How Do the U.S. Government's Decisions Affect Its Borrowing Costs?

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Abstract

A government's borrowing costs are encoded in the yield curve. We explore how the U.S. government's decisions on how much to spend and how much to tax affect the nominal yield curve.

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1 Introduction

Governments finance a substantial fraction of their outlays via debt issuance. While there is a large literature on how government decisions affect aggregate macroeconomic outcomes (for example, Romer and Romer (2010) and Mertens and Ravn (2013) study the effects of various tax changes, whereas Blanchard and Perotti (2002), Auerbach and Gorodnichenko (2012), Ramey (2011), and Ramey and Zubairy (2018) study the effects of government spending), there is surprisingly no work on the effects of government decisions on its borrowing costs as encoded in the yield curve of government liabilities.[1] Our paper tackles this question.

The question of how much a government's decisions change its borrowing costs is crucial for determining fiscal policies. This is most clearly evident from the literature on optimal fiscal policies in equilibrium models, where a government has to take into account how its actions will shift the yield curve (see, for example, Lucas and Stokey (1983), Barro (1979), and in particular models of optimal fiscal policy that explicitly incorporate the yield curve such as Buera and Nicolini (2004) and Angeletos (2002))[2].

Not only is there thus a theoretical motivation for studying the effects of fiscal policy on the yield curve, there is also indirect evidence that fiscal policy could have substantial effects on the yield curve. In particular, we are motivated by two sets of empirical findings: Ang and Piazzesi (2003) and Evans and Marshall (2007) highlight that macroeconomic factors are important drivers of the nominal yield curve. Furthermore, the literature on the macroeconomic impact of fiscal policy changes cited above generally finds substantial macroeconomic effects of fiscal policies.

To give one example of direct empirical evidence on the effect that government decisions have on the yield curve, Figure 1 plots the well-known series of exogenous tax changes identified by Romer and Romer (2010) in the top panel and then below scatter plots and the associated correlations between the shocks and various parts of the yield curve. The second row plots contemporaneous correlations whereas the bottom row plots the shock in quarter t and the yield curve in quarter $t+1$. We can see that there are significant correlations between these tax shocks and the yield curve, both contemporaneously and lagged.

[1] In the current paper we focus on the yields on nominal U.S. government debt.The reasons for this choice are threefold: First, the market on inflation-indexed bonds (TIPS) is substantially smaller and less liquid than the corresponding market for nominal debt, the time series on TIPS yields is much shorter, and, finally, the nominal yield curve in itself is a prominent object of study in economics and finance.

[2] While this point is clearly evident in models of *optimal* fiscal policy under *rational expectations*, knowledge about the effect of fiscal policies on prices are key ingredients in any model of fiscal policy - for a model of fiscal policy where policymakers do not have rational expectations see for example Karantounias (2020).

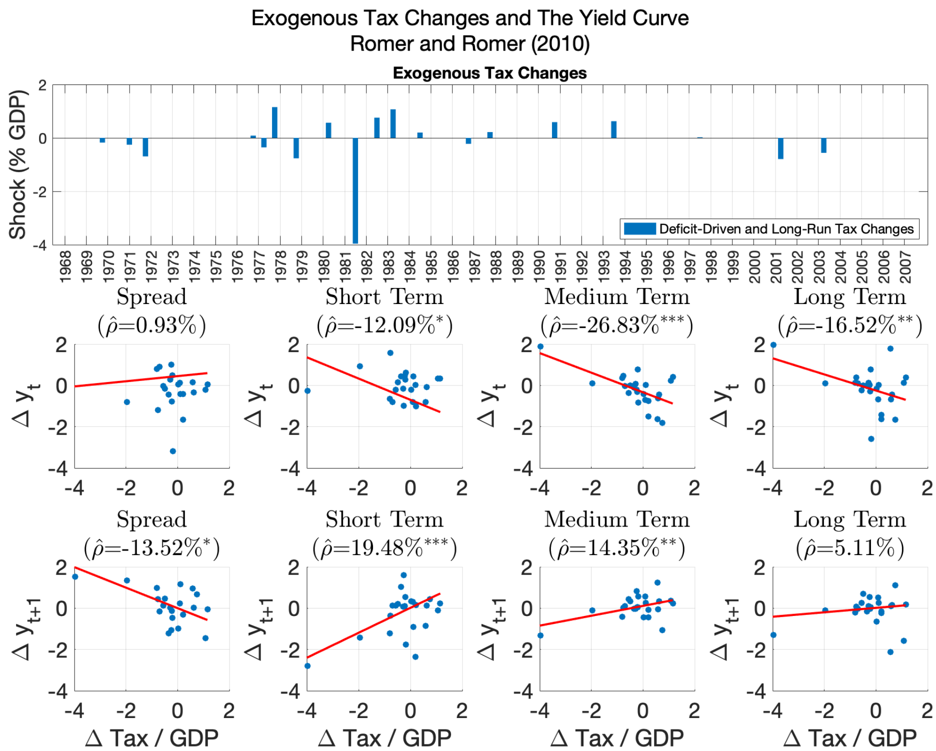

Figure 1: Correlations between government spending shocks and spread (difference between 10 year yield and 3 month yield), short (3 month yield), medium(5 year yield) and long term yields (10 year yield). The estimated correlation is in the title of each panel indicating the significance level $^{*}: 10\%, ^{**}: 5\%, ^{***}: 1\%$

Our paper pushes this type of analysis further by asking how various government actions affect not only certain points or functions of the yield curve, but the *entire* yield curve. We want to answer this question without imposing too much structure on the yield curve. We thus exploit recent advances in the theory of functional time series, where the yield curve at each point in time is viewed as a realization of a random functional stochastic process. With minimal structure imposed on the yield curve we can write this random function as a combination of countably many basis functions with time-varying weights. Furthermore, one can approximate this functional process well (in a sense we make precise later) using only a finite number of basis functions. This leaves us with only the task of tracking the finite-dimensional weights on these basis functions to characterize movements in the yield curve. We show how this approach can be cast as a state space model to aid interpretation. While this approach allows us to track movements in the yield curve, we want to go further and identify the *causal* link between a government's actions and changes in the yield curve. To do so, we borrow measures of *exogenous* variation (or shocks) to total government spending, defense spending, government consumption, and government investment from Auerbach and Gorodnichenko (2012) as well as shocks to personal and corporate income tax rates from Mertens and Ravn (2013).[3] The identification of these shocks is thus completely standard. We then estimate how these measures of policy changes are related to changes in the yield curve (the aforementioned weights in the basis functions, to be exact), which allows us to compute impulse responses of the entire yield curve to these policy changes.

In terms of related literature, the closest paper in terms of topic to ours is Berndt *et al.* (2012), who study the effects of defense spending shocks on the government's financing decision, i.e. whether the return on the government's portfolio changes after a defense shock or net surpluses change. Our paper focuses instead focuses on how different fiscal decisions affect nominal borrowing costs at different maturities. In terms of methodology, we borrow from the recent literature on functional time series analysis - see for example Chang *et al.* (2016)). The closest applied paper that uses ideas about estimating responses of entire functions to economic shocks is Inoue and Rossi (2019), who incorporate level, slope, and curvature factors from Nelson-Siegel type approach in a VAR to assess the effects of unconventional monetary shocks.

In the next section we use insights from the government budget constraint and an Euler equation to both further motivate our study and provide possible explanations for how government

[3]As robustness checks we also directly use the tax changes from Romer and Romer (2010) as well as the defense spending news shocks of Ramey (2011).

policies can influence the yield curve. In Section 3 we give an overview of our econometric methodology aimed at macroeconomists. After that we turn to our main results.

2 Two Concepts from Economic Theory

In this section we highlight two concepts that are helpful to motivate our analysis and to interpret the link between changes in fiscal policies and any associated changes in the yield curve for nominal government securities.

First, following Berndt *et al.* (2012), we analyze the government's budget constraint. In contrast to Berndt *et al.* (2012), we will analyze the *nominal* budget constraint because of our focus on the nominal yield curve. In nominal terms, the government's budget constraint is given by

$$B_{t+1} = R^b_{t+1}\left(B_t - S_t\right) \tag{1}$$

where B_t is the nominal value of outstanding government debt at the beginning of period t, S_t is the nominal primary surplus, and R^b_{t+1} is the nominal gross return on the government's portfolio between t and $t+1$ [4]. Directly borrowing from Berndt *et al.* (2012), the government budget constraint can be approximated via log-linearization as follows:[5]

$$ns_t - b_t = E_t \sum_{j=1}^{\infty} \rho^j \left(r^b_{t+j} - \Delta ns_{t+j}\right) \tag{2}$$

where ns_t is the weighted log nominal primary surplus ratio (for our purposes it will suffice to think of it as a measure of nominal surpluses), $b_t = \log B_t$, $r^b_t = \log R^b_t$, and ρ is a parameter between 0 and 1.

The key insight for our analysis is that changes in the surplus-to-debt ratio $ns_t - b_t$ will have to manifest themselves in changes in expectations of either (i) returns on the government portfolio or (ii) net surpluses. Berndt *et al.* (2012) focus on tracing out how changes in defense spending affects this decomposition. Our focus is broader: We ask how changes in *different* fiscal policies such as changes in different components of government spending and changes in different tax rates affect the government's borrowing costs. While these costs are

[4] Hall and Sargent (2011) have made substantial progress in computing theory- consistent measures of R^b_{t+1}. Other papers that have used the government budget constraint to analyze fiscal policy include Hilscher *et al.* (2014), Giannitsarou and Scott (2008), and Chung and Leeper (2007).

[5] One can verify that the analogous conditions derived by Berndt *et al.* (2012) for their log-linearization of the *real* budget constraint also hold for our log-linearization of the nominal counterpart.

encoded in r_t^b (see Hall and Sargent (2011) for a clear exposition), we want to disentangle how borrowing costs change across maturities - i.e. we directly study the effects of fiscal policies on the *entire* yield curve.

While the government budget constraint provides a direct link between changes in fiscal policies and borrowing costs, further information on the impact of fiscal policies on the yield curve can be gleamed using the insight that government securities have to priced in such a fashion as to entice market participants to purchase these securities.

To analyze this angle further, we turn to standard intertemporal asset pricing (see for example Cochrane (2001) and Campbell (2017)). In particular, we assume the existence of a positive real stochastic discount factor $\mathcal{M}_t$ (which might not be unique). We will now study the yield of a (zero-coupon) nominal government bond that matures next period. Such a bond pays a nominal return $R^n_{t,t+1}$ which is known at time t. We can use the stochastic discount factor to determine the yield via

$$1 = E_t\left(\mathcal{M}_{t+1}\frac{R^n_{t,t+1}}{\pi_{t,t+1}}\right)$$

where π denotes (gross) inflation.

Given that the yield is known at time t, we get that

$$\frac{1}{R^n_{t,t+1}} = E_t\left(\mathcal{M}_{t+1}\frac{1}{\pi_{t,t+1}}\right)$$

Next, we turn to multi-period risk-free nominal bonds (which deliver a known nominal return $R_{t,t+j}$ in j periods). Note that for zero coupon bonds, the yield is just $R^{1/j}_{t,t+j}$.

To price these assets, we define a multi-period stochastic discount factor as

$$\mathbb{M}_{t+j} = \prod_{t=1}^{t+j}\mathcal{M}_t$$

We can then price a nominally risk-free j period asset as

$$\mathbb{M}_t = R^n_{t,t+j}E_t\left(\mathbb{M}_{t+j}\frac{1}{\pi_{t,t+j}}\right)$$

For convenience, we define a third stochastic discount factor:

$$\mathbb{M}^*_{t+j} = \frac{\mathbb{M}_{t+j}}{\mathbb{M}_t}$$

We can then rewrite the equation above as

$$1 = R^n_{t,t+j} E_t \left(\mathbb{M}^*_{t+j} \frac{1}{\pi_{t,t+j}} \right)$$

Re-arrangeing this equation yields

$$\frac{1}{R^n_{t,t+j}} = cov_t \left(\mathbb{M}^*_{t+j}, \frac{1}{\pi_{t,t+j}} \right) + E_t(\mathbb{M}^*_{t+j}) E_t \left(\frac{1}{\pi_{t,t+j}} \right) = cov_t \left(\mathbb{M}^*_{t+j}, \frac{1}{\pi_{t,t+j}} \right) + \frac{1}{R_{t,t+j}} E_t \left(\frac{1}{\pi_{t,t+j}} \right)$$

where $R_{t,t+j}$ is the j-period return on a risk-free real asset[6]. We can use this equation to identify important drivers of the nominal yield curve. Note that the terms on the right-hand side of the previous equation are not independent, so shocks could move all objects on the right-hand side. Both the levels of the real-interest rate and expected inflation as well as the covariance between the inverse of inflation and the stochastic discount factor can be important. In particular, we now know that if a fiscal shock moves the nominal yield curve and in particular $R^n_{t,t+j}$, such a shock has to move either expectations of the (inverse of) inflation and real returns or the comovement between inflation and the stochastic discount factor. To interpret this comovement, we find it useful to impose more structure on the stochastic discount factor $\mathcal{M}$.

For illustrative purposes, we find it useful to make a strong assumption on $\mathcal{M}$: We use the stochastic discount factor based on the consumption Euler equation for log utility.[7] In that case we get

$$\mathcal{M}_{t+1} = \beta \frac{C_t}{C_{t+1}}$$

This tells us that an investor with log-utility really cares about states of the world where consumption growth is low.[8] In term of the earlier decomposition, the key covariance term on the right-hand side now becomes

$$cov_t \left(\beta^j \frac{C_t}{C_{t+j}}, \frac{1}{\pi_{t,t+j}} \right)$$

What we can take away from this analysis is that fiscal policy induced changed in nominal yields must make investors either update their views on average real returns (which are

[6]The inverse of this return is equal to the expected j period stochastic discount factor.

[7]Unfortunately log utility does not fit assets prices well generally, but it is useful to gain intuition.

[8]The risk free real return on a j-period security with log utility is given by $\left[E_t \left(\beta^j \frac{C_t}{C_{t+j}} \right) \right]^{-1}$.

directly linked to real consumption growth with this specific stochastic discount factor) and average inflation or the comovement between inflation and real consumption growth growth.[9] In particular, changes in fiscal policies could change investors' views of the government and thus lead them to update their perceptions of future economic growth and/or future inflation, an argument we will return to later.

3 A Hitchhiker's Guide to Functional Time Series Methods

In this section, we give a high-level overview of the functional time series methodology (Chang *et al.* (2016)) we use throughout our paper.[10] When large amounts of data are available on economic variables that are theoretically linked via a functional relationship (such as in various nominal yields being linked via the yield curve) our approach can directly exploit this functional relationship.

We assume that observations of the yield curve are generated by a function $y_t : I \to \mathbb{R}$ that describes the yield curve in the whole term structure. In period t, the yield for a security that matures in $t + \tau$ is given by $y_t(\tau)$ where τ is a value taken from the set I. Here, I is the interval of possible maturities (between 1 year and 30 years in our case).

3.1 Restrictions on the Yield Curve

In order to econometrically exploit the fact that all yields are linked via the yield curve, we will put one mild restriction on the yield curve. We only study yield curves that are in $\mathcal{L}^2(I)$, the space of square integrable functions. This space contains all functions $f(x)$ for which the integral over I of the square of $f(x)$ is bounded. While this space of functions is very general (it includes functions that are not continuous, for example), it still imposes some regularity, which seems reasonable for US data. More importantly, the restriction to $\mathcal{L}^2(I)$ gives us important mathematical tools to study the yield curve, which we discuss below. In particular, we can now define an inner product in $\mathcal{L}^2(I)$: For f and g we obtain

$$\langle f, g \rangle = \int_I f(x) \cdot g(x) dx \tag{3}$$

[9] Even with richer stochastic discount factors such as those derived using Epstein-Zin utility, consumption growth is still a key determinant - see for example Campbell (2017).

[10] More details are provided in the appendix.

A space that allow us to define a scalar product in it, like (3), is called a Hilbert Space. Not only is $\mathcal{L}^2(I)$ a Hilbert space, it is also a separable Hilbert Space[11]. This is a key property to analyze the yield curve because it allow us to write the yield curve as the linear combination of countable many functions $\{v_i(\tau)\}_{i=1,2,3,\cdots}$:

$$y_t(\tau) = \sum_{i=1}^{\infty} \alpha_{it} v_i(\tau) \tag{4}$$

Note that the functions $v_i(\tau)$ are independent of t and that we can describe $y_t(\tau)$ by the sequence of real numbers $\{\alpha_{1t}, \alpha_{2t}, \cdots\}$. To be precise, as long as the basis functions $v_i(\tau)$ are linearly independent, there is an isomorphic relationship between the space of functions and the space of all sequences $\mathbb{R} \times \mathbb{R} \times \mathbb{R} \cdots = \mathbb{R}^\infty$. This means, in our case, that every yield curve can be interpreted as sequence of real numbers and every sequence of real numbers can be traced back to a yield curve by combining the basis functions $v_1(\tau), v_2(\tau), \cdots$. Any set of linear independent functions in the space H will generate such an isomorphism, which leaves us with the choice of basis functions. We will pick our basis functions trough functional principal components that results from sequentially choosing each $v_i(\tau)$ for $i = 1, 2, ..m$ so that the largest amount of variability is explained by the sub-basis $v_1(\tau), v_2(\tau), v_3(\tau), \cdots, v_m(\tau)$.
This fact is quite remarkable as it implies that assuming the continuity of the yield curve and that the square of yield curve is integrable[12] allows us to study the complete term structure of the yield curve, not only modeling the cross-sectional interdependence of the different maturities but precisely exploiting it.
This approach is different than models of the yield curve that describe the level, slope and shape (or something similar): We are not imposing a particular set of functions to describe the yield curve - instead we choose basis functions that jointly describe most of the fluctuations in the yield curve.

3.1.1 A Finite Dimensional Representation of Yield Curve Dynamics

The dimension of a space is given by the number of elements in its basis. By this logic, the space H is infinite dimensional as the basis $\{v_i(\tau)\}_{i=1,2,3,\dots}$ that we used in (4) has infinitely many elements. But we may consider "smaller" spaces, such as the space H_m that consists of all elements that can be written as the linear combination of the first m elements of the bases $\{v_1(\tau), v_2(\tau), \cdots, v_m(\tau)\}$. Thus, H_m is an m-dimensional space even if the elements in

[11] A separable Hilbert Space have bases that are countable, $v_1, v_2, v_3, ...$
[12] This is not too restrictive as the square of the yield curve will likely be bounded.

H_m are functions, which are infinite dimensional.
The function $y_t(\tau)$ is not an element of H_m given that we need more than just the first m elements of the basis to represent it as we can see in (4). We can consider the projection of $y_t(\tau)$ on H_m given by

$$\tilde{y}_t(\tau) = \sum_{i=1}^{m} \alpha_{it} v_i(\tau) \tag{5}$$

This gives us an equation akin to an observation equation in a state space model

$$y_t(\tau) = \sum_{i=1}^{m} \alpha_{it} v_i(\tau) + w_t(\tau) \tag{6}$$

where $w_t(\tau) = y_t(\tau) - \tilde{y}_t(\tau)$ is the approximation error we make by restricting ourselves to H_m.
Let us now introduce a mapping from H_m to $\mathbb{R}^m$

$$H_m \ni \tilde{y}_t(\tau) \mapsto \alpha_t = \begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \vdots \\ \alpha_{mt} \end{pmatrix} \in \mathbb{R}^m$$

Note that with the basis $\{v_1(\tau), v_2(\tau), \cdots, v_m(\tau)\}$ and $\begin{pmatrix} \alpha_{1t} & \alpha_{2t} & \cdots & \alpha_{mt} \end{pmatrix}'$ we can recover $\tilde{y}_t(\tau)$ through (5). This isometry allow us to analyze an infinite dimensional object $y_t(\tau)$ through a finite dimensional approximation $\alpha_t \in \mathbb{R}^m$.
Using functional principal components (as discussed in the appendix) we determine a basis of functions $\{v_i(\tau)\}_{i=1,2,3,\ldots}$ such that its first m elements generate $y_t^{(m)} \in \mathbb{R}^m$ a "best" approximation of $y_t(\tau)$.[13] Note that we can thus effectively choose a very efficient set of basis functions for our purposes instead restricting ourselves to an a-priori chosen basis function such as the monomials $\{1, \tau, \tau^2 \ldots\}$.[14] In particular, this approach to selecting basis functions makes makes dealing with non-stationarities in the yield curve possible.
For the yield curve, with $m = 5$ we can express over 99.99% of the variability of $y_t(\tau)$. This principal components analysis (detailed in the appendix) also delivers a time series for the

[13]Given the way that the yield curve is constructed our basis functions are effectively linear combinations of so-called wavelets, as discussed in the appendix).
[14]We show a plot of the first three basis function we use below in Figure 2.

vector of weights $\alpha_t = \begin{pmatrix}\alpha_{1t} & \alpha_{2t} & \cdots & \alpha_{mt}\end{pmatrix}'$. This vector α_t can't be directly interpreted as yields as the measurement equation highlights that only together with the basis functions $\{v_1(\tau), v_2(\tau), \cdots, v_m(\tau)\}$ we can recover the yield curve. It does, however, serve as the state in our state-space model for the yield curve.[15] We next posit a VAR law of motion for α_t. In particular, we focus on a VAR(1) for the sake of parsimony.

$$\alpha_t = A\alpha_{t-1} + u_t \tag{7}$$

From an applied perspective, our approach can be thought of as modeling observations on the yield curve at each point in time t via a state-space framework with a set of observation equations (see equation 6) that link the yield of an asset with a specific maturity to a set of basis functions that depend on the maturity and weights on each basis function, which vary over time, but do not depend on maturity. These weights represent the states in our state space model, which we model as a Vector Autoregression (VAR). as in equation 7.
It turns out that in our application the matrix A will have unit roots. We therefore find it useful to rewrite the VAR for the state variables in error correction form:

$$\alpha_t = \alpha_{t-1} + \gamma\beta'\alpha_{t-1} + u_t \tag{8}$$

Recall that in an error correction model β is the matrix of cointegrating relationships in α_t. Given that α_t was constructed by principal components, if the system has ℓ unit-roots, then the $(\ell+1)^{th}, (\ell+2)^{th}, \cdots, m^{th}$ components have been stripped from all the unit roots so

[15] The analogy to state space models might be slightly misleading because we first compute the states via principal components and then go on to model the law of motion for the estimated states, whereas standard applications of state space models often employ a filtering algorithm (think about the Kalman Filter, for example) that exploits a posited law of motion for the states when estimating the states. Our approach is instead very much reminiscent of the standard two-step approach to linear factor models in standard time series analyses (see for example Stock and Watson (2016)). The resulting model of the yield curve is still of the state-space form.

each one of them individually is stationary. Therefore, we can fix β to be

$$\beta = \begin{pmatrix} 0_{\ell \times (m-\ell)} \\ I_{m-\ell} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ & \vdots & \\ 1 & 0 & 0 \\ & \vdots & \\ 0 & 0 & 1 \end{pmatrix}$$

α_t and its associated basis functions have a very clear interpretation in our application, as we highlight in Figure (2), which plots the basis functions associated with the first three elements of α_t. Note that the first two basis functions are associated with elements of α_t that have unit roots. We show two versions of our basis functions: one where we explicitly impose that the first basis function is a constant, and one where we leave the first basis function unconstrained. It turns out that the first basis functions is basically estimated to be a constant anyway, but for clarity we impose this constraint from now on.
This means that one of the unit root processes will permanently shift the entire yield curve by the same amount each period independently of maturity, while the second component will permanently tilt the yield curve.

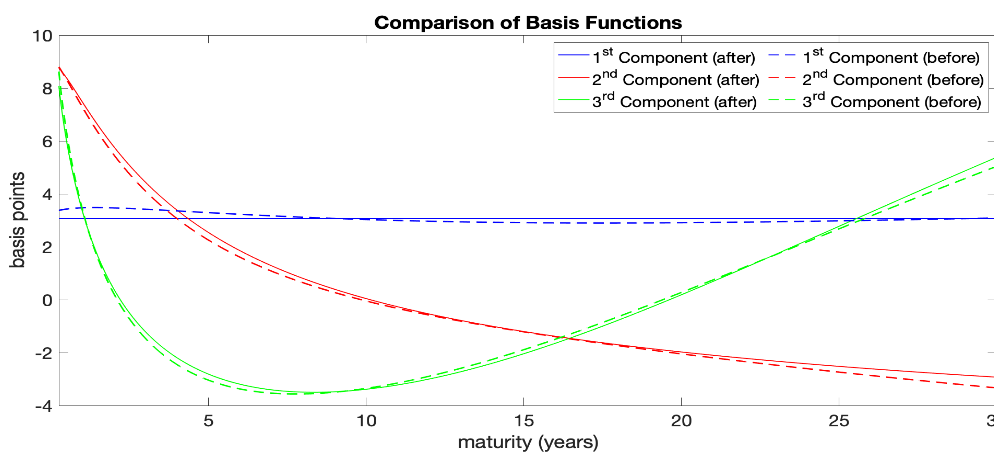

Figure 2: Basis functions before and after imposing the restriction of the first component to be the constant function.

3.2 Identifying Shocks

Our goal is to assess how α_t changes as government policies change (as measured by the various fiscal policy shocks we use). To do so we proceed as follows: First, we estimate values of the one-step ahead forecast errors u_t. We find it useful to rotate these shocks so that they can be directly interpreted as either temporary shocks or permanent shocks. This helps us with the interpretation of the effects of fiscal shocks, because we will be able to link the fiscal shocks to either one of two permanent shocks (described below) or a temporary shock. Then we will regress our measures of the fiscal shocks on these three shocks driving the yield curve. The resulting regression coefficients will help us determine the impulse responses of α_t (and hence ultimately the yield curve) to fiscal shocks.
Where do these shocks come from? It turns out that there are two permanent shocks because we find two unit roots in equation (7). Adding one temporary shock means these three shocks explain 98.48% of the overall variance of u_t in equation (7).[16] We thus use two layers of approximation in our analysis: First, we use a finite dimensional approximation of the infinite dimensional vector α_t. And then we focus on the three most prevalent shocks in u_t (our approximation will thus make the covariance matrix of the approximation to u_t have rank 3. We will call these three shock "semi-structural" from now on.

3.3 Identification Strategy for Three "Semi-Structural" Shocks

We aim to identify three semi-structural shocks:

1. *Transitory Shock:* A shock with a response that has *no permanent effects*. Given that the permanent space H_P is two dimensional, identification of a transitory shock implies two restrictions.

2. *Level Shock:* A permanent shock with a response that has no *permanent spread effects*. Because the permanent spread space is one dimensional, this identification implies *one restriction*. Note that because of the form of the basis functions outlined in Figure (2), this shock will naturally be associated with the first element of α_t.

3. *Spread Shock:* A permanent shock with a response that may have both level and spread longrun effects. Given the shape of the basis functions outlined in Figure (2), this shock

[16] Technically, the sum of the three largest eigenvalues of u_t is equal to 98.48% of the total sum of all five eigenvalues.

will naturally be associated with both the first and second elements of α_t. There are no restrictions for identification of this shock.

We define ϵ^{lev}, ϵ^{spd} and ϵ^{tra} to be the *level*, *spread* and *transitory* shocks to the yield curve.

3.4 Impulse Responses to Fiscal Shocks

Given our previous discussions, we are in a position to compute impulse responses to the semi-structural shocks outlined above. All that is left to do is to link these responses to observable measures of fiscal shocks. To do so, we regress each fiscal shock measure ϵ_t^e on the three semi-structural shocks: [17]

$$\hat{\epsilon}_t^e = \beta_{lev}\epsilon_t^{lev} + \beta_{spd}\epsilon_t^{spd} + \beta_{tra}\epsilon_t^{tra} \tag{9}$$

The impact response of α_t to a one standard deviation shock in ϵ_t^e is then given by

$$\beta_{lev}r^{lev} + \beta_{spd}r^{spd} + \beta_{tra}r^{tra}$$

where r^x is the impact response of α_t to semi-structural shock x.

4 Results

For the yield curve we use the model introduced by Gürkaynak *et al.* (2007) the data can be obtained in the Board of Governors' website[18]. In our sample for the yield curve starts in 1961. We use quarterly data - in particular, we use the yield curve on the last day of each quarter as or quarterly observaiton of the yield curve. The exact sample for the regression of the fiscal shocks on the yield curve shocks depends on the availability of the various shocks - the samples for those regressions can be found in the original sources.

We measure various government spending shocks by identified shock from a linear VAR as

[17] We show the R^2 for each regression in Appendix D.

[18] https://www.federalreserve.gov/data/nominal-yield-curve.htm

described by Auerbach and Gorodnichenko (2012).[19] Throughout, we will present impulse responses for the yield curve by plotting how the entire yield curve changes t periods after a shock. We will refer to t as the *horizon*, while we refer to the different *maturities* of the yield curve to distinguish them from the horizon of the impulse response.

4.1 Shocks to Government Spending and Its Components

4.1.1 Shocks to Total Government Spending

Figure 3 shows the impulse response of the yield curve to an unexpected change in overall government spending. There is no significant change at the short end of the yield curve. Yields at long maturities increase significantly and do so permanently (i.e. this government spending shock co-moves with the permanent semi-structural shocks we have identified). The consumption highlights various avenues through which these changes can occur: (i) permanent changes in the conditional expectations of (the inverses of) consumption growth and inflation or its covariance. A change in government spending can influence these objects in various ways: an unexpected change in fiscal policy could indeed just raise future consumption directly through standard macroeconomic channels, but if imperfect information is important then changes in government spending could also provide signals about either the preferences or information of the government, which could lead households, firms, and markets to update expectations.

[19] One slight deviation from that paper is that we control for forecasts of overall government spending in *all* our VAR specifications. We do so to make sure that our identified shocks are truly unforecastable (Auerbach and Gorodnichenko (2012) do not do this for all specifications). However, it turns out that the impact of this change is minimal - the results are very similar if we take the exact specifications from Auerbach and Gorodnichenko (2012), as we show in Appendix C.

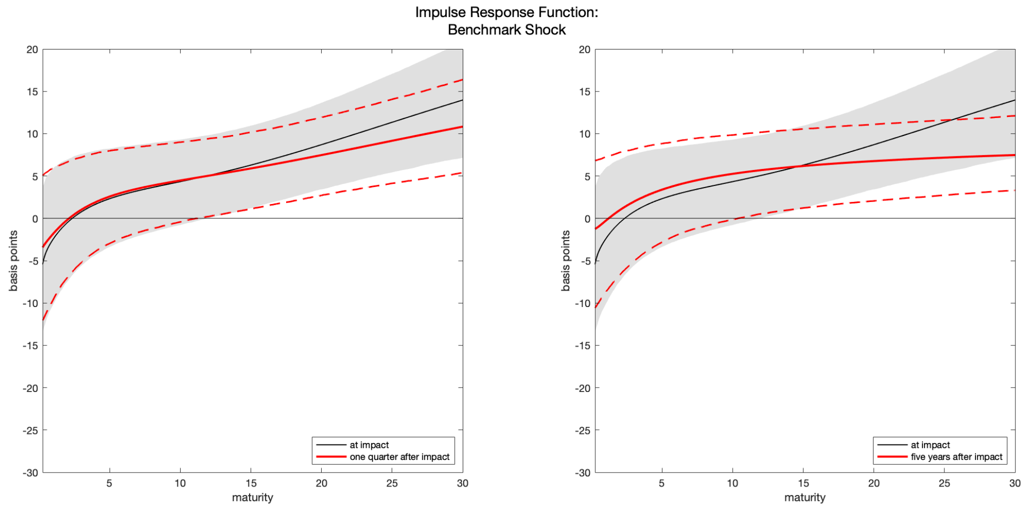

Figure 3: Impulse responses of the yield curve to a one standard deviation Government-spending shock. Error bands are 68% significance bands computed using our bootstrap procedure.

4.1.2 Shocks to Defense Spending

We next turn to shocks to defense spending. These types of shocks have become prominent in macro because many changes in defense spending are arguably exogenous with respect to macroeconomic conditions (Ramey (2011)). [20] We again follow Auerbach and Gorodnichenko (2012) to identify defense spending shocks. As Figure 4 shows, we find no significant movements in the yield curve after defense spending shocks. This results is robust to using the defense news shocks from Ramey (2011) as our shock measure instead, as we show in the appendix.

[20]Disadvantages of this approach are that effects of defense spending might not be representative of the effects of broader government spending, and unexpected variation in government spending might be dominated by few events so that there is generally not a lot of variation that can be exploited in empirical studies. We therefore chose to directly interpret our results as effects of defense spending shocks instead of interpreting them as effects of general government spending.

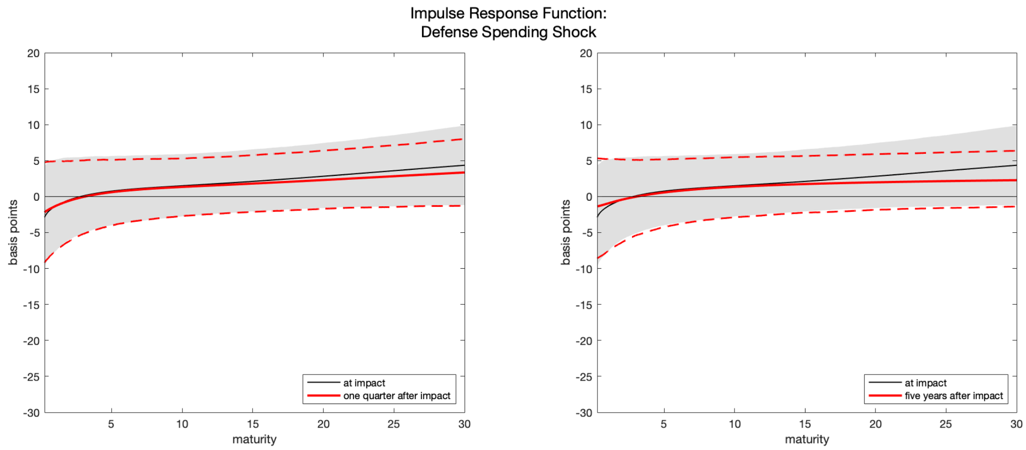

Figure 4: Impulse responses of the yield curve to a one standard deviation Defense spending shock. Error bands are 68% significance bands computed using our bootstrap procedure.

If we instead use the defense spending shock as developed in Ramey (2011) we find very similar results - see Appendix B.1 for the corresponding impulse responses.

4.1.3 Shocks to Government consumption and Investment

As highlighted by Boehm (2019), effects of government spending can vary substantially depending on whether the shocks are government *consumption* or government *investment* shocks. We identify shocks to these components of government spending as in Auerbach and Gorodnichenko (2012). Figures 5 and 6 show that these shocks indeed have very different effects on the yield curve. There is a significant negative response of the yield curve to a consumption shock. Such a shock drags down the yield curve across all maturities on impact (as can be seen in the left panel) and these effects are to a large extent persistent. A Shock to consumption spending could lead to a decrease in yields because it signals that the economy is not doing well, thus reducing expected growth. Furthermore, such a reasoning could lead the central bank to cut interest rates, which can move the yield curve down.
The investment shock has a very different impact: it shifts the yield curve up. This effect is not significant at the short end of the yield curve for all horizons we consider, but becomes significant at even slightly longer maturities. This could represent the investor's view that government investment will lead to permanent positive effects on growth.[21]

[21] Note that after five years our point estimate of the impulse response is basically flat across maturities.

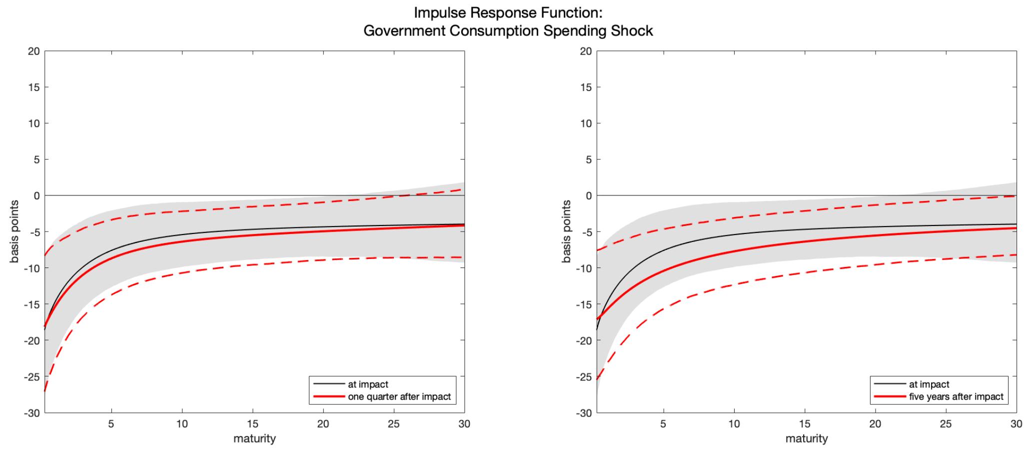

Figure 5: Impulse responses of the yield curve to a one standard deviation Government Consumption spending shock. Error bands are 68% significance bands computed using our bootstrap procedure.

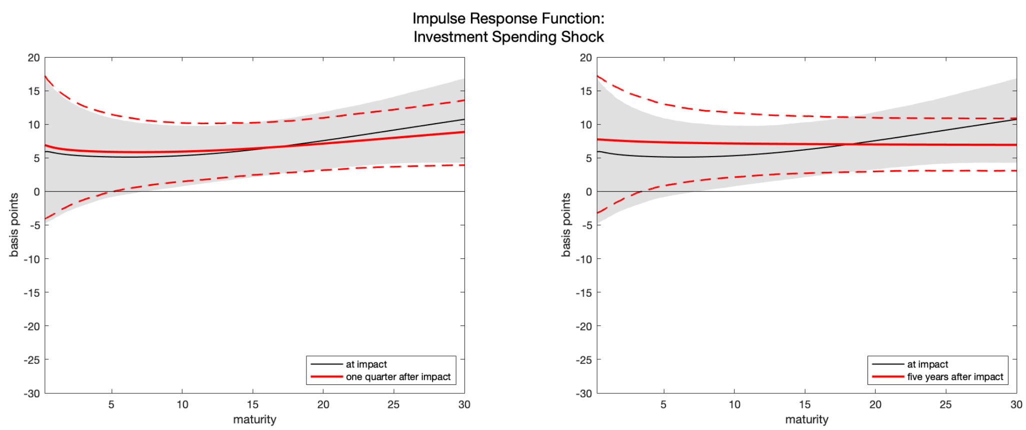

Figure 6: Impulse responses of the yield curve to a one standard deviation Government Investment spending shock. Error bands are 68% significance bands computed using our bootstrap procedure.

4.2 Income Taxes

We next turn the effects of unexpected changes in corporate and private income tax rates. Our measures of shocks to these rates is borrowed from Mertens and Ravn (2013). Throughout we normalize shocks (as in Mertens and Ravn (2013)) so that a shock *increases* the relevant tax rate on impact.

4.2.1 Personal Income Taxes

Personal income taxes have a strong effect at short maturities (in fact the largest absolute effect on short maturities across all shocks we study). As highlighted by Mertens and Ravn (2013), these shocks have sizeable but temporary effects on GDP growth, which could explain the initial increase at short maturities. This effect is persistent though, as can be seen from the right panel of Figure 7. When viewed through the lens of the Euler equation, these results could suggest that market participants view these changes as strongly and positively affecting consumption growth.

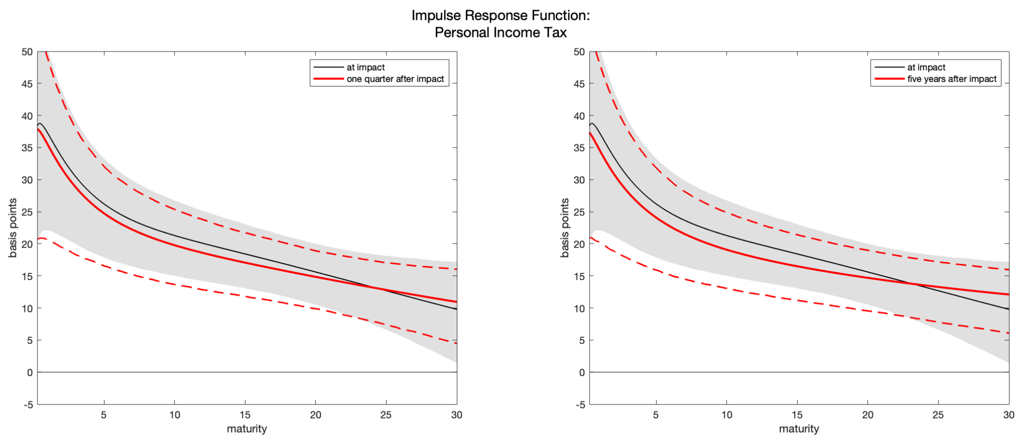

Figure 7: Impulse responses of the yield curve to a one standard deviation Personal Income Tax shock. Error bands are 68% significance bands computed using our bootstrap procedure.

4.2.2 Corporate Income Taxes

Corporate tax rate changes turn out to have very different effects on the yield curve compared to personal income tax changes. A corporate income tax increase persistently lowers the yield curve across the board. Mertens and Ravn (2013) find that the output effects of these shocks are smaller than for personal tax changes (but still positive and significant). One possible explanation is that market participants view such a corporate tax change as a signal of lower future growth. The absolute magnitudes are smaller than for the personal tax changes though. As we show in Appendix B.2, the results when we directly use the Romer and Romer (2010) measure as a tax shock are very similar to the impulse responses obtained using the corprate tax shock from Mertens and Ravn (2013).

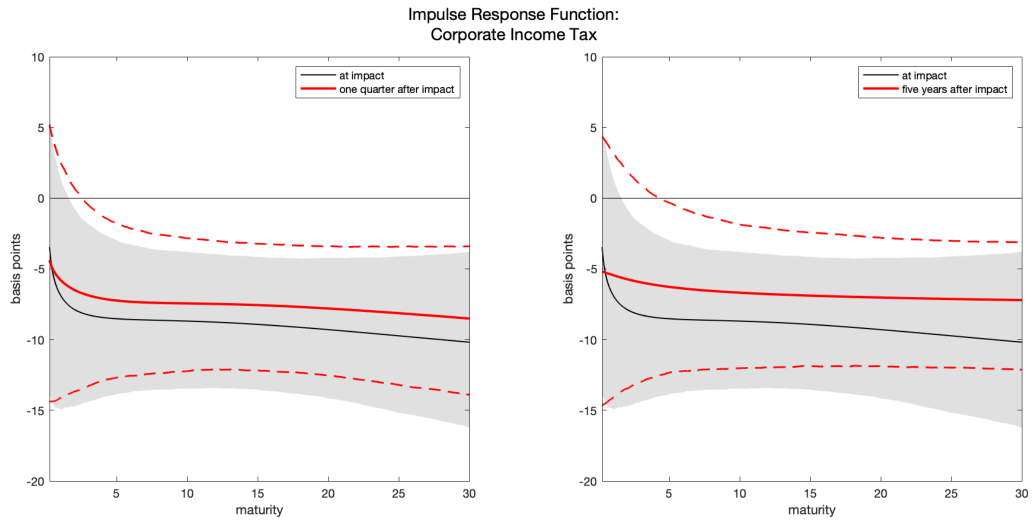

Figure 8: Impulse responses of the yield curve to a one standard deviation Government Corporate Income Tax shock. Error bands are 68% significance bands computed using our bootstrap procedure.

5 Conclusion

We have used recent advances in functional time series modeling to assess how the yield curve moves after changes in fiscal policy. To be able to make causal statements we have borrowed time series of unexpected changes (or shocks) in various fiscal instruments from the rich literature on macroeconomic effects of fiscal policies.

Our key findings are that (i) there are strong very persistent/ permanent effects of all fiscal policies (except defense spending) on the yield curve owing to the two unit roots found in the nominal yield curve, and (ii) in terms of magnitudes one-standard deviation shocks to personal income tax rates have the largest effects on the yield curve.

References

Ang, A. and Piazzesi, M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics*, **50** (4), 745–787.

Angeletos, G.-M. (2002). Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure. *The Quarterly Journal of Economics*, **117** (3), 1105–1131.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the Output Responses to Fiscal Policy. *American Economic Journal: Economic Policy*, **4** (2), 1–27.

Barro, R. J. (1979). On the Determination of the Public Debt. *Journal of Political Economy*, **87** (5), 940–971.

Berndt, A., Lustig, H., and Yeltekin, S. (2012). How Does the US Government Finance Fiscal Shocks? *American Economic Journal: Macroeconomics*, **4** (1), 69–104.

Blanchard, O. and Perotti, R. (2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. *The Quarterly Journal of Economics*, **117** (4), 1329–1368.

Boehm, C. E. (2019). Government consumption and investment: Does the composition of purchases affect the multiplier? *Journal of Monetary Economics*.

Buera, F. and Nicolini, J. P. (2004). Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics*, **51** (3), 531–554.

Campbell, J. (2017). *Financial Decisions and Markets: A Course in Asset Pricing*. Princeton University Press.

Chang, Y., Kim, C. S. and Park, J. Y. (2016). Nonstationarity in time series of state densities. *Journal of Econometrics*, **192** (1), 152 – 167.

Chung, H. and Leeper, E. (2007). *What Has Financed Government Debt?* CAEPR Working Papers 2007-015, Center for Applied Economics and Policy Research, Department of Economics, Indiana University Bloomington.

Cochrane, J. (2001). *Asset Pricing*. Princeton University Press.

Daubechies, I. (1992). *Ten lectures on wavelets*, vol. 61. Siam.

Evans, C. L. and Marshall, D. A. (2007). Economic determinants of the nominal treasury yield curve. *Journal of Monetary Economics*, **54** (7), 1986–2003.

Giannitsarou, C. and Scott, A. (2008). Inflation Implications of Rising Government Debt. In *NBER International Seminar on Macroeconomics 2006*, NBER Chapters, National Bureau of Economic Research, Inc, pp. 393–442.

Gürkaynak, R. S., Sack, B. and Wright, J. H. (2007). The us treasury yield curve: 1961 to the present. *Journal of Monetary Economics*, **54** (8), 2291–2304.

Hall, G. J. and Sargent, T. J. (2011). Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics. *American Economic Journal: Macroeconomics*, **3** (3), 192–214.

Hilscher, J., Raviv, A. and Reis, R. (2014). *Inflating Away the Public Debt? An Empirical Assessment*. NBER Working Papers 20339, National Bureau of Economic Research, Inc.

Inoue, A. and Rossi, B. (2019). *The Effects of Conventional and Unconventional Monetary Policy: A New Approach*. Working Papers 1082, Barcelona Graduate School of Economics.

Karantounias, A. (2020). *Doubts about the model and optimal policy*. Tech. rep., Federal Reserve Bank of Atlanta.

Lucas, R. J. and Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, **12** (1), 55–93.

Mertens, K. and Ravn, M. O. (2013). The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States. *American Economic Review*, **103** (4), 1212–1247.

Ramey, V. A. (2011). Identifying Government Spending Shocks: It's all in the Timing. *The Quarterly Journal of Economics*, **126** (1), 1–50.

— and Zubairy, S. (2018). Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data. *Journal of Political Economy*, **126** (2), 850–901.

Romer, C. D. and Romer, D. H. (2010). The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks. *American Economic Review*, **100** (3), 763–801.

Stock, J. and Watson, M. (2016). Dynamic Factor Models, Factor-Augmented Vector Autoregressions, and Structural Vector Autoregressions in Macroeconomics. In J. B. Taylor and H. Uhlig (eds.), *Handbook of Macroeconomics*, *Handbook of Macroeconomics*, vol. 2, *0*, Elsevier, pp. 415–525.

A How to Model the Yield Curve Computationally?

In Gürkaynak *et al.* (2007) they build a model for the yield curve that interpolates the information available to obtain an estimate of the actual yield curve at some period t that matures in τ years :

$$y_t(\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{\tau}{\phi_1}}}{\frac{\tau}{\phi_1}} + \beta_2 \left[\frac{1 - e^{-\frac{\tau}{\phi_1}}}{\frac{\tau}{\phi_1}} - e^{\frac{-\tau}{\phi_1}} \right] + \beta_3 \left[\frac{1 - e^{-\frac{\tau}{\phi_2}}}{\frac{\tau}{\phi_2}} - e^{\frac{-\tau}{\phi_2}} \right] \tag{10}$$

about their estimation Gürkaynak *et al.* (2007) state in their paper: we choose the parameters to minimize the weighted sum of the squared deviations between the actual prices of Treasury securities and the predicted prices.

Let us for example consider a term structure starting at 3 months ($\tau = 0.25$) and ending in 10 years ($\tau = 30$), computationally, we consider 1024 maturities from $\tau_1 = 0.25$ to $\tau_{1024} = 30$ in equally separated increments. With the values of the yield curve in those maturities we obtain the observation in time t of the yield:

$$y_t^\delta = (y_t(\tau_1), y_t(\tau_2), \cdots, y_t(\tau_{1024})) \in \mathbb{R}^{1024}$$

There are two reasons to look for an alternative way of computing the yield curve: On the one had, the information in y_t^δ is not efficiently presented as there is a degree of redundancy in the value of the components, it is certain that the value of $y_t(\tau_n)$ is very close to and highly correlated with $y_t(\tau_{n+1})$, it would be better if every component would provide us with as much information about the yield curve as possible not included already in other component. The second, rather technical reason, is that the basis implicitly meant in y_t^δ is based in so-called Dirac measures, these are functions that have the value one for τ_n and 0 elsewhere in I

$$\delta_{\tau_n}(x) = \begin{cases} 1 & \text{if } x = \tau_n \\ 0 & \text{elsewhere in } I \end{cases}$$

these functions are not continuous functions and therefore not functions in $\mathcal{C}(I)$.
More appropriately, we consider a basis of functions called "wavelets" and using a transformation we obtain from

$$\mathbb{R}^{1024} \ni y_t^\delta \mapsto y_t^w \in \mathbb{R}^{1037}$$

see (Daubechies, 1992) for more details on the exact implementation of the wavelets.
A sample from the basis functions used to obtain y_t^w is shown in Figure 9

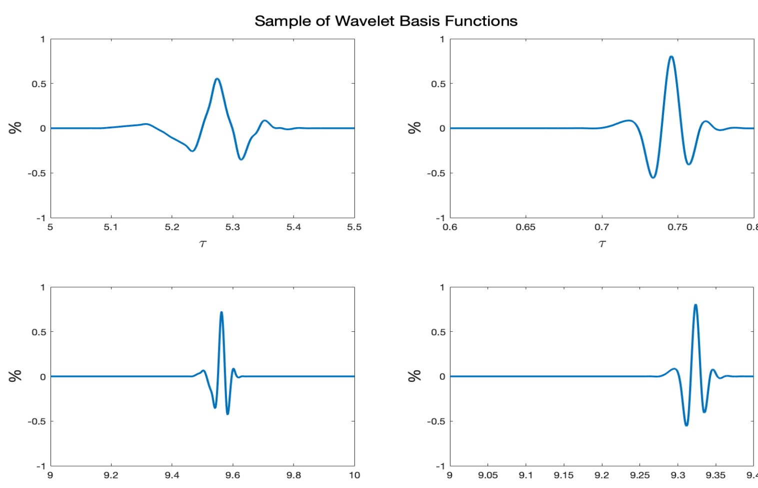

Figure 9: Selected basis functions of the wavelet basis.

Wavelets combine the frequency domain and time domain to optimally represent data (Daubechies, 1992). The combination of the wavelet basis and the sample of $\mathbb{R}^{1037}$ vectors $\{y_t^w\}$ are the computational representation of the yield curve that we use for further analysis. Using wavelets here allow us to stick to our functional interpretation of the data, as the basis functions are continous functions while we center our analysis on the weights or coefficients of those functions given in $\{y_t^w\}$.

A.1 What is the "best" H_m?

Now we explain how to apply functional principal components to determine, given m, which is the optimal space H_m that best represents the variability y_t in an $m-$dimensional space. We consider each observation of y_t a random function, that means, we assume the observed value of the yield curve is the realization of a random process. The distribution of y_t, as an infinite dimensional object, is certainly an abstract concept that requires several technical details that go beyond the scope of this note. Nevertheless, we are interested in at least understanding what is the expectation of a random function $\mathbb{E}y_t$ and the covariance (operator[22]) of a random function.

Given the random function y_t, in a separable Hilbert space H, we consider a fixed function $u \in H$ and the scalar product $\langle u, y_t \rangle \in \mathbb{R}$. Not only this is a scalar, by definition of scalar product, but also, even when u is fixed, this is a random variable due to the randomness of y_t and as a scalar random variable it has an expectation: $\mathbb{E}\langle u, y_t \rangle$, we are familiar with the notion of the expectation of a scalar random variable. This object, as a matter of fact, can be interpreted as what mathematicians call a "linear functional"

$$H \ni u \mapsto \mathbb{E}\langle u, y_t \rangle \in \mathbb{R} \tag{11}$$

the linearity follows from the properties of the scalar product and the expectation operator. Obviously, this linear functional depends on y_t, actually, there are many of such linear functional. A very useful theorem in Functional Analysis actually tells us that the space of linear functionals defined in H and the space H, itself, are isomorphic. This means that, for every linear functional $T : H \to \mathbb{R}$ there is one element of H that corresponds to it. The space of all such T's is called the dual space of H and it is typically represented with H^*. For any T_w in H^* there is $w \in H$ such that

$$T_w(u) = \langle u, w \rangle$$

Thus, for the linear functional defined in (11) there is one corresponding element in H that we define to be $\mathbb{E}y_t \in H$.

For two centered[23] random functions f and g in H we define $\mathbb{E}(f \otimes g)$ to be their covariance

[22]It is helpful to think of an operator as the infinite dimensional generalization of a matrix.

[23]By centered we mean that $\mathbb{E}f = \mathbb{E}g = 0$. Where here 0 is the constant function 0.

operator such that for all u, w in H

$$\langle u, \mathbb{E}(f \otimes g)w \rangle = \mathbb{E}\langle u, f \rangle \overline{\langle v, g \rangle} \tag{12}$$

Note that the expectation (12) gives us a way to compute the the covariance operator. The expectation on the left is the expectation of the product of two scalar random variables and should be intuitively clear.
Let us assume now that y_t is a centered process. We define the covariance operator Q to be $Q = \mathbb{E}y_t \otimes y_t$. The spectrum of Q is the set of real numbers $|Q - \lambda \cdot 1|$ is singular. Here 1 represents the identity operator such that $1(u) = u$. Given the fact that H is a separable Hilbert space the spectrum of Q is countable. The elements in the spectrum Q are ordered as follows: $\lambda_1 \geq \lambda_2 \geq \cdots$ for each one of those values we have eigenfunctions that solve

$$(Q - \lambda_i \cdot 1)v_i = 0$$

We consider the basis $\{v_1, v_2, v_3, \cdots, v_m\}$ which are elements of H and therefore continuous functions $v_i : I \to \mathbb{R}$. The portion of the variance of y_t in H that is captured by $y_t^{(m)}$ in H_m is given by the ratio

$$\frac{\sum_{i=1}^{m} \lambda_i}{\sum_{i=1}^{\infty} \lambda_i}$$

by considering the values λ_i in decending order we are capturing the largest portion possible by a basis of m elements.

Estimation of H_m

Once we have the sample of the yield curve $\{y_t^w\}_{t=1,\cdots,T}$ given in terms of the basis of wavelet functions we proceed to obtain what we referred to as "optimal" basis at the beginning of this note.
To estimte H_m we need to estimate Q first. We use y_t^w in the following way to obtain the estimator of Q

$$\hat{Q} = T^{-1} \sum_{i=1}^{T} y_t^w y_t^{w'}$$

recall that y_t^w is a vector and that the outer-product is well defined here. We approximate $v_1, v_2, \cdots$ with the eigenvectors of $\hat{Q}$: $\hat{v}_1, \hat{v}_2, \cdots$ that correspond to the eigenvalues of $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \cdots$ and we estimate the portion of variance explained by the first m eigenvalues by

$$\frac{\sum_{i=1}^{m} \hat{\lambda}_i}{\sum_{i=1}^{1037} \hat{\lambda}_i} \tag{13}$$

to choose an appropriate m we want the expression of (13) to reach some threshold such a 90% for example. Values of m "too" large can cause an phenomenon called illed-posed problem: λ_m decreases to a point that Q is close to singular. We expected this to happen as the number of parameter used to generate y_t in (10) as even when the parameters do no interact linearly the sources of variations is finite.
The space H_m is then estimate by span$(\{v_1, v_2, \cdots v_m\})$

B Additional Specifications for Robustness

Here we show results from alternative specifications used to corroborate our results.

B.1 Narrative Government Spending Shock

(Ramey, 2011)

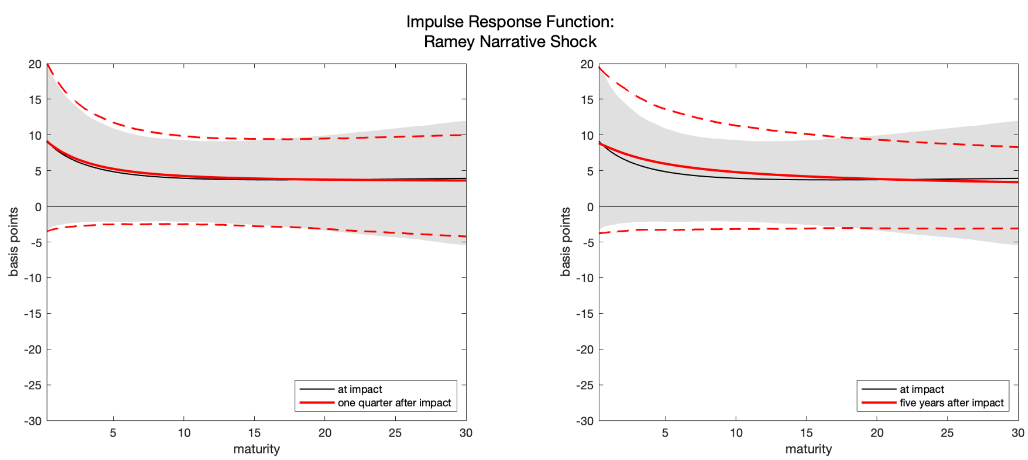

Figure 10: Impulse responses of the yield curve to a narrative government spending shock. Error bands are 68% significance bands computed using our bootstrap procedure.

B.2 Narrative Tax Shock

Romer and Romer (2010)

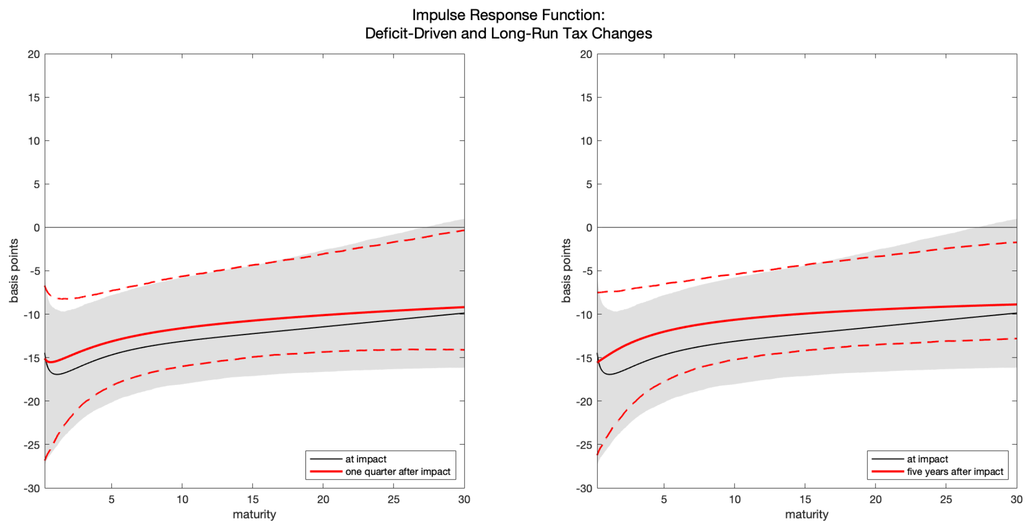

Figure 11: Impulse responses of the yield curve to a narrative tax shock as estimated in Romer and Romer (2010). Error bands are 68% significance bands computed using our bootstrap procedure.

C Adding Forecasts of Government Spending to Our VARs

As highlighted in the main text, we add the forecasts of total government spending growth from the Survey of Professional Forecasters to all our VARs for government spending. In this section we show that the resulting shocks are actually very similar to sticking to the original specifications from Auerbach and Gorodnichenko (2012). For the sake of brevity we only show plots of the two resulting shock series for each government spending shock we consider.

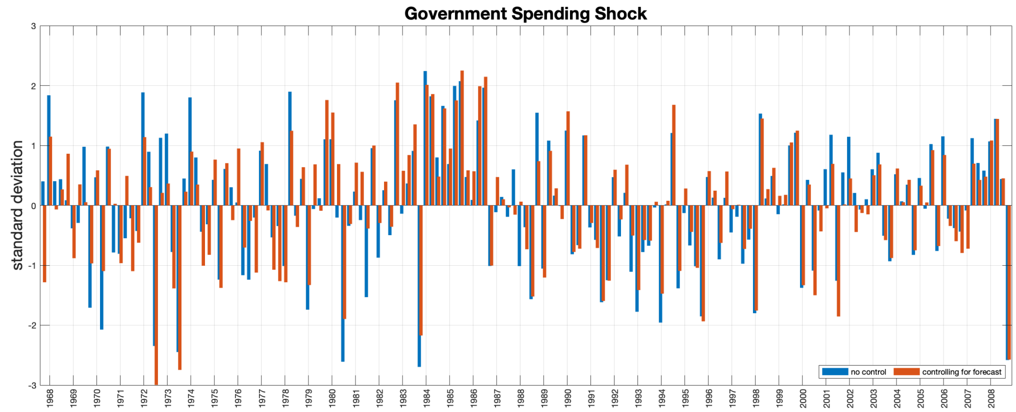

Figure 12: Government spending shocks with and without controlling for expectations.

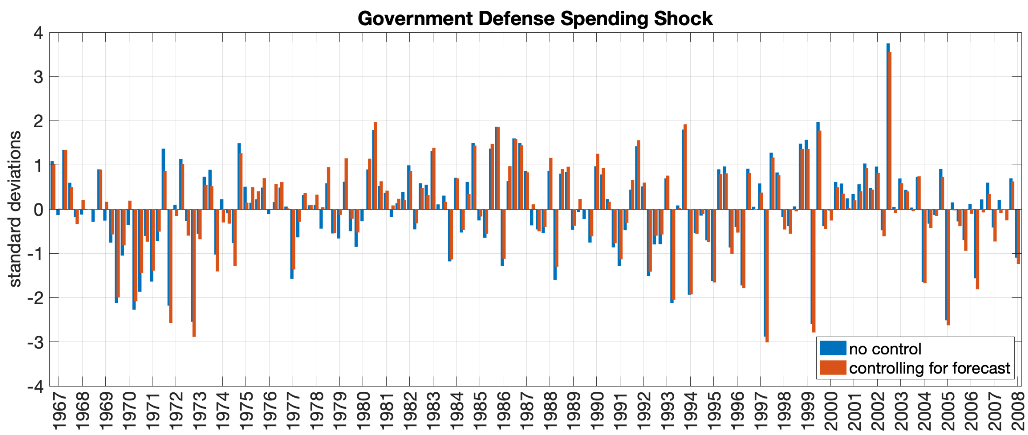

Figure 13: Defense spending shocks with and without controlling for expectations.

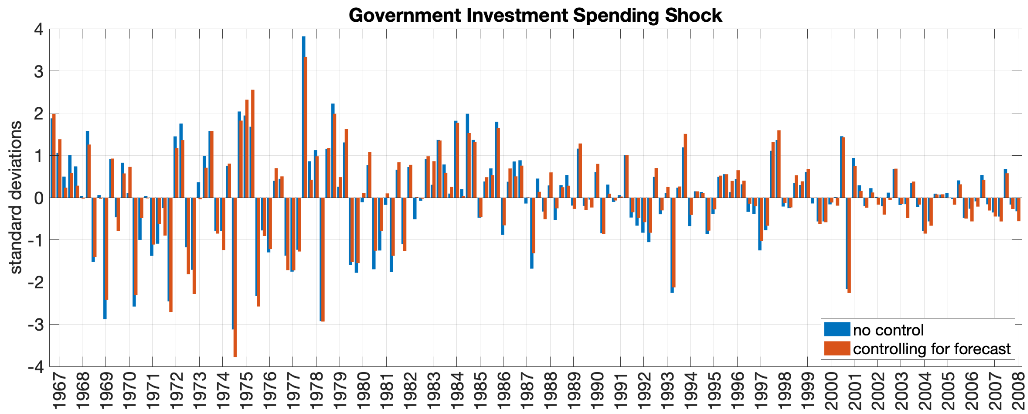

Figure 14: Investment spending shocks with and without controlling for expectations.

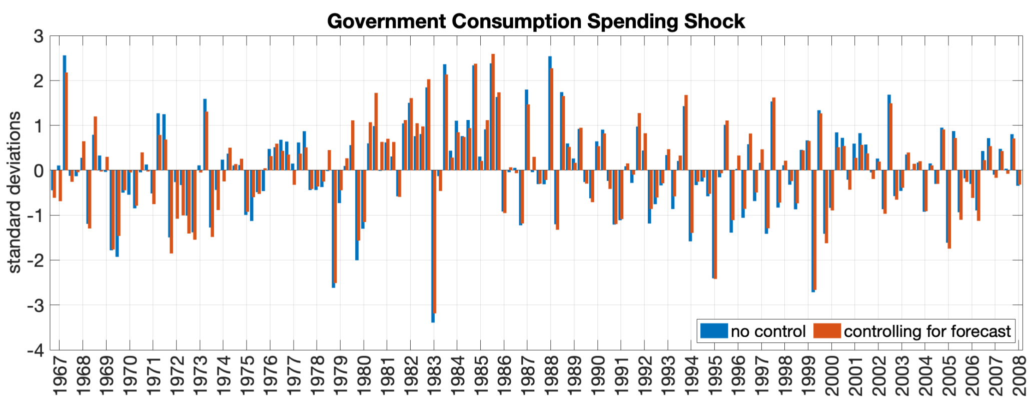

Figure 15: Consumption spending shocks with and without controlling for expectations.

D R^2 for Regressions Linking Fiscal Shocks and Yield Curve Shocks

Shock	**R^2 in equation (9)**
Benchmark Shock	5.66% [2.44%,8.87%]
Defense Shock	1.94% [0.54%,3.34%]
Non-Defense Shock	3.15% [0.96%,5.33%]
Consumption Shock	4.33% [1.74%,6.92%]
Investment Shock	4.16% [1.5%,6.82%]

Table 1: R^2 and boostrap-based 16th and 84th percentiles.

E Bootstrapping

We will use bootstrapping to determine the confidence intervals of the statistics we estimate for the yield curve and its responses to external shocks. In the case we are interested in determine the response of the yield curve to an external shock we consider a sample of the yield curve that matches the sample of the external shock. The majority of the external shocks we consider here are given in quarterly frequency so we consider the last daily observation of the yield curve on the corresponding quarter. That way, we have two identical samples in size and frequency for the yield curve and an external shock.

In the following, we show how to generate copies of functional time series and the external shock of size n:

1. Take the residuals from (8): $\hat{u}_t = \Delta y_t^{(m)} - \hat{\alpha}\beta y_{t-1}^{(m)}$
2. Pick, with replacement, a sample of n residuals $\hat{u}_{b(1)}, \hat{u}_{b(2)}, \cdots, \hat{u}_{b(n)}$ from $\{\hat{u}_1, \hat{u}_2, \cdots\}$
3. If an interaction with an external shock ϵ_t is of interest, the bootstrap copy of the external shock becomes $\epsilon_{b(1)}, \epsilon_{b(2)}, \cdots, \epsilon_{b(n)}$. This is to assure the preservation of contemporaneous effects of the external shock and the generated series.
4. Define the new set of residuals $u_t^* = \hat{u}_{b(t)} - \mu_{u_b}$, where μ_{u_b} is the mean of the sample taken in step 2. The mean of u_t^* is therefore 0.

5. Generate a time series $y_t^{(m)*}$ by $y_t^{(m)*} = \hat{A}_m y_{t-1}^{(m)*} + u_t^*$ using $y_0^{(m)*} = y_0^{(m)}$.

6. Use the basis $\{\hat{v}_1, \hat{v}_2, \cdots, \hat{v}_m\}$ and the time series $y_t^{(m)^*}$ to generate a copy of the functional time series y_t^*.

7. Use the demeaned functional time series y_t^* and if applies the shocks from step 3 to estimate the statistics to be analyzed.

In this study, we estimate the responses at different horizons of the yield curve to the semi-structural shocks identified above. The correlation between the semi-structural shocks and the external shocks. The regression coefficients and R^2 of the fiscal policy shock on the semi-structural shocks. The impulse response function of the yield curve to the projection of the fiscal policy shock on the semi-structural shocks and the forecast error variance decomposition. We calculate each of these statistics for every copy $\tilde{y}_t$ and determine the confidence intervals from the quantiles of each calculated statistic.