A Multivariate Endogenous Regime-Switching SVAR: Exchange Rate Pass-Through in Costa Rica

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Abstract

We develop an endogenous regime-switching structural VAR (RS-SVAR) that extends the framework of Chang, Choi and Park (2017) from univariate autoregressive models to a multivariate setting. The model allows identified structural shocks to influence both macroeconomic outcomes and regime transition probabilities, enabling a disciplined decomposition of nonlinear transmission mechanisms into their structural sources. We apply the framework to study exchange rate pass-through (ERPT) in Costa Rica, a small open economy where exchange rate fluctuations play a central role in inflation dynamics.

Using monthly data from 2009 to 2025, we find that measured pass-through elasticities are broadly similar across regimes once responses are scaled by the exchange rate movement induced by the identified shock. The apparent variation in raw pass-through measures reflects state-dependent exchange rate dynamics rather than shifts in pricing behavior — regimes while exchange rate movements differ considerably in magnitude. We further document that unexpected inflation shocks systematically alter the probability of regime transitions, a finding that cannot be obtained from models with exogenous switching. These results illustrate how embedding endogenous regime-switching within a structural VAR provides a framework capable of distinguishing between competing sources of nonlinear macroeconomic transmission.

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Key Words: exchange rate pass-through, endogenous regime-switching, structural VAR, small open economy

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1 Introduction

Understanding how exchange rate movements transmit to domestic prices is central to both inflation forecasting and monetary policy design in small open economies. The exchange rate pass-through effect (ERPT) is, *ceteris paribus*, the response of domestic prices to variations in the nominal exchange rate. Despite its importance, empirical estimates of ERPT vary considerably across studies, specifications, and time periods — a dispersion that is difficult to reconcile with a single stable elasticity and that motivates the search for state-dependent transmission mechanisms.

Figure 1 presents a simple descriptive exercise for Costa Rica: the ratio between contemporaneous changes in monthly inflation and changes in the exchange rate, which we refer to as a "naive" measure of pass-through. Because this ratio becomes mechanically unstable when exchange rate changes are small, it should be interpreted as purely descriptive. Nonetheless, the figure highlights two stylized facts. First, the implied pass-through ratio exhibits substantial dispersion over time, with values ranging from well below zero to above 100%. Second, this dispersion does not reveal a clear systematic pattern — neither the direction of the exchange rate movement (appreciation versus depreciation) nor its magnitude appears to reliably predict the implied pass-through ratio. These observations suggest that a single stable transmission coefficient is an inadequate description of exchange rate pass-through dynamics, and motivate the search for state-dependent mechanisms that go beyond sign or size asymmetries.

This paper develops an econometric framework designed to address this challenge. We propose an endogenous regime-switching structural VAR that generalizes the framework of Chang, Choi and Park (2017), originally developed for univariate autoregressive processes, to a multivariate VAR setting. The model allows identified structural shocks to influence both macroeconomic outcomes and regime transition probabilities — a feature that distinguishes it from conventional Markov-switching models with exogenous and time-invariant transition probabilities (Hamilton, 1989; Krolzig, 1997). By embedding regime switching directly into the structural system, the framework enables a disciplined decomposition of nonlinear transmission mechanisms into their structural sources.

The framework makes two distinct contributions relative to existing approaches. First, by separately identifying the structural parameters governing the exchange rate's own response and the contemporaneous inflation response across regimes, the model can decompose the ERPT ratio into its numerator and denominator components — a decomposition that is not

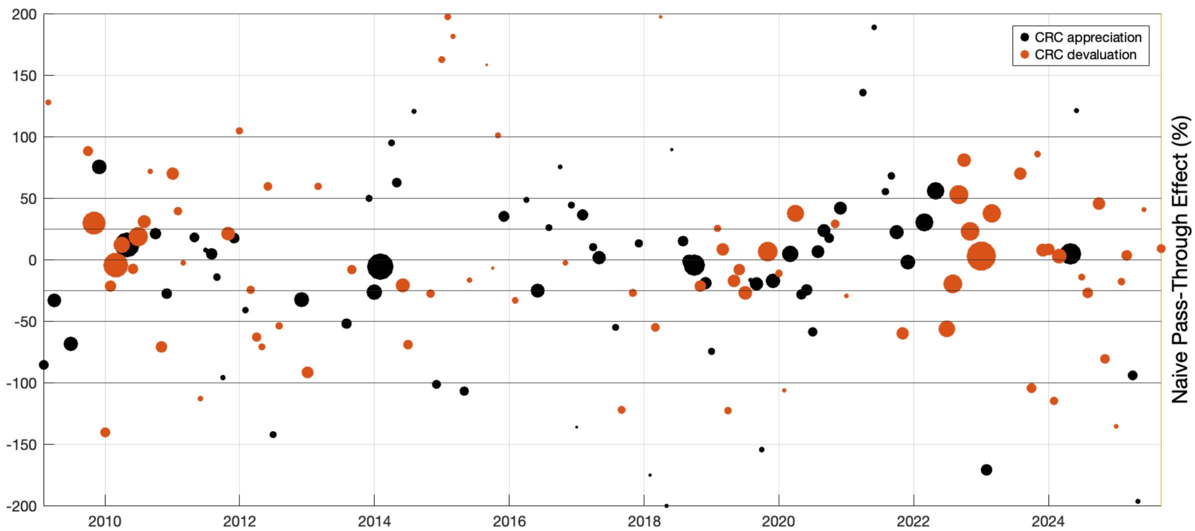


Figure 1: Ratio of contemporaneous changes in inflation $\Delta\pi$ and changes in the exchange rate $\Delta\%EXR_t$. The color of the dot represents positive (black) or negative (red) exchange rate changes. The size of the dot represents the absolute magnitude of the exchange rate variation. The blue lines indicate the range of previous pass-through estimates. Sample: 02.2009–10.2025.

recoverable from a heteroskedastic SVAR that only allows shock variances to shift across states. Second, the endogenous feedback from identified structural shocks to regime transition probabilities provides information about regime dynamics that is entirely absent from variance-switching models. Together these features allow the researcher to determine not only whether pass-through differs across regimes, but *why* it differs — whether through shifts in pricing behavior, changes in exchange rate volatility, or feedback from macroeconomic disturbances to regime dynamics.

We apply the framework to study exchange rate pass-through in Costa Rica, a small open economy where the USD–CRC exchange rate exhibits recurrent episodes of volatility and where price adjustment is subject to nominal rigidities and institutional frictions. The structural VAR framework (Guillermo Peón and Rodríguez Brindis, 2014; Forbes *et al.*, 2018) enables us to isolate exchange rate innovations orthogonal to external cost pressures and global financial conditions, and to trace their effects on inflation across regimes.

Our main findings are threefold. First, we identify two volatility regimes with distinct exchange rate dynamics. Cumulative pass-through reaches approximately 20% at a one-year horizon in both regimes, with confidence bands overlapping substantially — indicating that pass-through elasticities are not statistically distinguishable across states. The apparent variation in raw pass-through measures instead reflects state-dependent exchange rate dynamics: inflation responses to identified exchange rate shocks are stable across regimes while exchange rate movements differ considerably in magnitude. Variation in the denominator of the pass-through ratio, rather than in pricing behavior, drives the observed asymmetries.

Second, regime transitions are endogenously driven by domestic nominal disturbances. Unexpected inflation shocks increase the probability of elevated exchange rate volatility, consistent with a feedback mechanism in which inflation surprises erode confidence in the nominal anchor and generate subsequent exchange rate pressure. Idiosyncratic exchange rate shocks, by contrast, are associated with subsequent stabilization — consistent with mean-reverting bilateral rate dynamics or a policy response by the central bank. External commodity price shocks do not directly drive regime transitions.

Third, robustness exercises using alternative information sets, recursive orderings, and sample periods yield qualitatively similar conclusions, supporting the reliability of the identified transmission mechanisms.

These findings carry implications for inflation forecasting and policy in small open economies. The stability of pass-through elasticities across regimes suggests that apparent variation in measured pass-through need not reflect shifts in firm pricing behavior, but may instead reflect differences in the magnitude of exchange rate disturbances. Disentangling these sources requires a framework capable of separately identifying exchange rate dynamics and inflation responses — precisely the decomposition our model provides.

Related literature. Early studies on exchange rate pass-through in Costa Rica include León *et al.* (2001), who report short-run pass-through of 16% and long-run effects up to 55%, and Castrillo and Laverde (2007), who find smaller short-run effects. Rodríguez (2009) documents a negative relationship between exchange rate volatility and pass-through, a finding our framework helps rationalize by attributing apparent pass-through variation to the volatility of the exchange rate response rather than to shifts in pricing behavior. More recent work emphasizes nonlinearities: Esquivel and Gomez-Rodriguez (2010) employ a logistic smooth transition VAR, while Brenes and Esquivel (2017) document asymmetric responses between depreciations and appreciations. Our contribution complements this literature by modelling regime variation as an endogenous outcome within a structural multivariate framework, and by providing a decomposition of the sources of apparent nonlinearity.

Outline. Section 2 develops the econometric framework, presenting the baseline structural VAR, the regime-dependent impact structure, the endogenous regime transition mechanism, the likelihood and filtering algorithm, and identification. Section 3 presents the empirical application to Costa Rica, including the linear benchmark, regime-dependent transmission results, and the endogenous regime dynamics. Section 4 reports robustness checks across alternative information sets, recursive orderings, and sample periods. Section 5 discusses broader implications of the framework for pass-through analysis and small open economy modelling. Section 6 concludes.

2 A Multivariate Endogenous Regime-Switching SVAR

This section develops a multivariate structural vector autoregression (SVAR) framework with endogenous regime switching. The model extends a standard linear SVAR by allowing selected elements of the structural impact matrix and the regime-transition probabilities to vary across latent states. This flexibility permits nonlinearities to arise either through regime-dependent transmission mechanisms or through regime-dependent shock intensity.

The framework builds on three components. First, a baseline structural VAR captures the dynamic interactions among exchange rates, inflation, and other macroeconomic variables. Second, the contemporaneous impact structure is allowed to differ across regimes, enabling state-dependent impulse responses. Third, regime transitions are endogenous: the probability of switching between regimes depends on observable economic conditions and past shocks.

This structure provides a disciplined way to distinguish between two conceptually distinct sources of nonlinear behavior. Regime dependence may reflect changes in structural transmission elasticities—such as shifts in firms' pricing behavior—or it may capture variations in the magnitude and volatility of shocks. By embedding regime switching directly into the structural system, the model allows the data to determine which of these mechanisms drives observed asymmetries.

The following subsections describe the baseline linear specification, the regime-dependent impact structure, the endogenous transition mechanism, and the economic interpretation of the resulting regimes.

2.1 Baseline Structural VAR

We begin with a standard structural vector autoregression (SVAR),

$$y_t = c + \sum_{i=1}^{p} \Phi_i y_{t-i} + u_t, \tag{1}$$

where y_t is a d-dimensional vector of endogenous variables that includes, at a minimum, the exchange rate and inflation. The vector c collects intercept terms, Φ_i are autoregressive coefficient matrices, and u_t denotes reduced-form residuals. We assume $u_t \sim \mathbb{N}(0, \Sigma)$.[1]

Our objective is to isolate an identified exchange rate innovation and trace its effect on inflation in order to measure exchange rate pass-through effect. Exchange rate fluctuations reflect multiple underlying shocks—external demand, commodity prices, financial conditions, and domestic disturbances. A structural VAR framework separates these components by imposing identification restrictions that recover orthogonal structural innovations from reduced-form residuals. This reduces spurious co-movements between inflation and the exchange rate driven by common shocks. The use of SVAR models to estimate pass-through is standard in the literature (see, *inter alia*, Guillermo Peón and Rodríguez Brindis, 2014, Forbes *et al.*, 2018, Ha *et al.*, 2020).

Structural shocks are obtained by factorizing the reduced-form residuals:

$$u_t = B\varepsilon_t, \qquad \text{with} \quad \Sigma = BB', \tag{2}$$

where ε_t denotes mutually orthogonal structural shocks satisfying

$$\mathbb{E}[\varepsilon_t \varepsilon_t'] = I_d.$$

The matrix B is not unique: if B satisfies (2), then BH also satisfies it for any orthogonal matrix H.[2] To achieve identification, we impose recursive restrictions and assume B is lower triangular.

Under this identification scheme, each column of B describes the contemporaneous response of all variables to a specific structural shock. Suppose, for concreteness, that the

[1] Gaussianity is not required for consistent estimation of a linear VAR but facilitates likelihood-based estimation in the regime-switching extension.

[2] An orthogonal matrix H satisfies $HH' = I$.

exchange rate is the third variable in y_t and inflation the fourth. The third column of B,

$$B_{\cdot 3} = (b_{13}, b_{23}, b_{33}, b_{43})',$$

captures the impact effects of an exchange rate shock. The coefficient b_{33} measures the exchange rate's own response, while b_{43} measures the contemporaneous response of inflation. The impact pass-through elasticity is therefore given by the ratio b_{43}/b_{33}.

In the linear specification, this ratio is constant over time. The next subsection generalizes the model by allowing the impact structure to vary across regimes.

2.2 Regime-Dependent Impact Structure

To allow for nonlinear dynamics, we replace the time-invariant impact matrix B with a regime-dependent matrix,

$$B_t = B(s_t),$$

where $s_t \in \{0,1\}$ denotes an unobserved regime indicator. We interpret $s_t = 0$ as a low exchange-rate volatility regime and $s_t = 1$ as a high-volatility regime.

The regime-switching VAR is given by

$$y_t = c + \sum_{i=1}^{p} \Phi_i y_{t-i} + u_t, \tag{3}$$

$$u_t \sim \mathbb{N}(0, \Sigma(s_t)), \tag{4}$$

with $\Sigma(s_t) = B(s_t)B(s_t)'$.

Under recursive identification, the regime-dependent impact matrices take the form

$$u_t = B(0)\varepsilon_t = \begin{bmatrix} + & 0 & 0 & 0 \\ * & + & 0 & 0 \\ * & * & \boxed{\substack{+ \\ \text{(low)}}} & 0 \\ * & * & \boxed{*} & + \end{bmatrix} \varepsilon_t, \qquad u_t = B(1)\varepsilon_t = \begin{bmatrix} + & 0 & 0 & 0 \\ * & + & 0 & 0 \\ * & * & \boxed{\substack{+ \\ \text{(high)}}} & 0 \\ * & * & \boxed{*} & + \end{bmatrix} \varepsilon_t, \tag{5}$$

where $*$ denotes unrestricted coefficients and $+$ denotes parameters restricted to be positive. The boxed elements correspond to the exchange rate's own response and the contemporaneous response of inflation to an exchange rate shock.

Allowing B to vary with s_t permits regime dependence in both the magnitude of exchange rate movements and the associated inflation response. Importantly, this flexibility does not impose regime dependence on pass-through a priori; instead, it allows the data to determine whether nonlinearities arise through changes in transmission elasticities or through variations in shock amplitude.

This specification extends the endogenous regime-switching framework of Chang *et al.* (2017), originally developed for univariate autoregressive processes, to a multivariate structural VAR. By embedding regime switching directly into the structural system, the model allows regime transitions to be linked to identified shocks and macroeconomic conditions within a coherent multivariate environment.

2.3 Endogenous Regime Dynamics

Regime transitions are governed by a latent state variable w_t, assumed to follow an autoregressive process,

$$w_t = \alpha w_{t-1} + \eta_t, \tag{6}$$

where $|\alpha| < 1$ controls regime persistence and η_t is an innovation term. The observed regime indicator s_t is determined by a threshold rule:

$$s_t = \begin{cases} 0 & \text{if } w_t < \tau, \\ 1 & \text{if } w_t \geq \tau, \end{cases}$$

where τ is an unknown threshold parameter. Different combinations of α and τ generate flexible regime durations and transition frequencies.

Regime dynamics are *endogenous* in the sense that the probability of switching between regimes depends on the history of structural shocks. Specifically, we allow the innovation η_t to be correlated with lagged structural disturbances,

$$\rho = \mathrm{Corr}(\eta_t, \varepsilon_{t-1}),$$

so that past identified shocks influence the evolution of the latent state. This specification permits structural shocks to affect not only macroeconomic outcomes but also the likelihood of entering a different regime. At the same time, contemporaneous orthogonality is preserved, ensuring that identification of structural shocks remains intact.

Unlike exogenous Markov-switching models with constant transition probabilities, this framework allows regime probabilities to evolve as a function of identified macroeconomic disturbances.

The complete endogenous regime-switching SVAR consists of equations (3), (4), and (6), together with the parameters (α, τ, ρ). Estimation proceeds by maximum likelihood using the filtering approach of Chang *et al.* (2017), extended here to the multivariate structural VAR setting.

2.4 Conceptual Interpretation

The regime-switching structure is intentionally parsimonious. Rather than allowing the entire impact matrix to vary across regimes, we restrict regime dependence to the exchange-rate shock transmission block—specifically, the exchange rate's own response (b_{33}) and the contemporaneous response of inflation to an exchange rate shock (b_{43}). All other structural relationships remain constant across states.

This restriction focuses the nonlinear component of the model on exchange-rate dynamics and their transmission to prices. Regime dependence may therefore arise through two channels. First, the magnitude of exchange rate movements in response to an identified exchange rate shock may vary across regimes (via b_{33}), capturing differences in exchange-rate volatility or shock amplification. Second, the contemporaneous response of inflation to that shock may vary (via b_{43}), reflecting potential changes in pricing behavior.

Exchange rate pass-through at impact is given by the ratio b_{43}/b_{33}. If both coefficients vary proportionally across regimes, impulse responses will differ in scale while the pass-through elasticity remains stable. In contrast, regime-dependent pricing behavior would manifest as differences in this ratio across states.

By allowing regime variation only in the exchange-rate shock transmission block, the model isolates nonlinearities in pass-through dynamics while preserving a stable structural environment for the remaining macroeconomic relationships.

2.5 Likelihood and State Filtering

Estimation is conducted by maximum likelihood. Under Gaussian innovations, the joint process (s_t, u_t)—equivalently (s_t, ε_t)—admits a first-order Markov transition density. Chang *et al.* (2017) establish this result for a univariate model. Appendix A extends their argument to the present multivariate structural VAR and provides a closed-form expression for the regime transition probability.

Let θ collect all parameters:

$$\theta = \Big(c, \{\Phi_i\}_{i=1}^{p}, B(0), B(1), \alpha, \tau, \rho\Big).$$

Conditional on regime $s_t = j$, the reduced-form innovation satisfies

$$u_t \mid (s_t = j) \sim \mathcal{N}(0, \Sigma(j)), \qquad \Sigma(j) = B(j)B(j)'.$$

Let $f_j(u_t;\theta)$ denote the corresponding Gaussian density under regime j.

2.5.1 Transition Probabilities in Closed Form

The multivariate extension derived in Appendix A implies that the regime transition probability can be written as

$$\omega_\rho(\cdot) \equiv \mathbb{P}_\theta(s_t = 0 \mid s_{t-1}, \varepsilon_{t-1}),$$

where $\omega_\rho(\cdot)$ is available in closed form (involving $\Phi(\cdot)$ and $\varphi(\cdot)$) and depends on (α, τ, ρ) as well as lagged structural shocks.

This object introduces endogeneity into the regime dynamics: past structural shocks influence the probability of switching regimes, while contemporaneous identification remains governed by the impact matrix $B(s_t)$.

2.5.2 Filtering Algorithm

Define the information set $\mathcal{F}_{t-1} = \sigma(y_{t-1}, y_{t-2}, \ldots)$ and let

$$\pi_{t-1}(i) \equiv \mathbb{P}_\theta(s_{t-1} = i \mid \mathcal{F}_{t-1}), \qquad i \in \{0,1\}.$$

Given $\pi_{t-1}(i)$, form the predicted regime probabilities

$$\tilde{\pi}_t(j) = \sum_{i\in\{0,1\}} \mathbb{P}_\theta(s_t = j \mid s_{t-1} = i, \varepsilon_{t-1})\, \pi_{t-1}(i), \qquad j \in \{0,1\},$$

where $\mathbb{P}_\theta(s_t = j \mid s_{t-1} = i, \varepsilon_{t-1})$ is obtained from $\omega_\rho(\cdot)$.

Let

$$u_t(\theta) = y_t - c - \sum_{i=1}^{p} \Phi_i y_{t-i}$$

denote the reduced-form innovation implied by parameter vector θ. The one-step predictive density is the mixture

$$p_\theta(u_t \mid \mathcal{F}_{t-1}) = \sum_{j\in\{0,1\}} f_j\Big(u_t(\theta);\theta\Big)\, \tilde{\pi}_t(j).$$

Updating yields filtered probabilities

$$\pi_t(j) = \frac{f_j(u_t(\theta);\theta)\, \tilde{\pi}_t(j)}{\sum_{k\in\{0,1\}} f_k(u_t(\theta);\theta)\, \tilde{\pi}_t(k)}.$$

The log-likelihood function is then

$$\ell_T(\theta) = \sum_{t=1}^{T} \log p_\theta(u_t \mid \mathcal{F}_{t-1}).$$

The multivariate transition result established in the appendix provides the closed-form transition term needed to evaluate $p_\theta(u_t \mid \mathcal{F}_{t-1})$ and therefore the likelihood.

2.6 Identification and Interpretation

A key challenge in regime-switching SVARs is that regime identification is driven by the likelihood contribution of the reduced-form innovations. Under Gaussianity, the conditional density of u_t depends on the regime only through the regime-specific covariance matrix,

$$u_t \mid (s_t = j) \sim \mathcal{N}(0, \Sigma(j)), \qquad \Sigma(j) = B(j)B(j)'.$$

As a consequence, the likelihood is primarily informative about changes in $\Sigma(s_t)$, rather than changes in the structural impact matrix $B(s_t)$ per se. In particular, different structural parameterizations that imply similar covariance matrices may be difficult to distinguish in finite samples. This feature creates a fundamental tension: while the objective is to learn about regime dependence in structural transmission (e.g., pass-through elasticities), the likelihood tends to identify regimes most cleanly when they generate pronounced differences in the second moments of u_t.

In this sense, regime dependence in structural parameters can be weakly identified when it is observationally close to alternative parameterizations that imply similar $\Sigma(s_t)$.

This observation motivates a parsimonious regime-dependent structure. Rather than allowing many structural parameters to vary across regimes—which can exacerbate weak identification and increase the risk of empirically indistinguishable regimes—we restrict regime dependence to a small subset of coefficients with direct economic relevance. In our application, only the exchange rate's own response to an exchange rate shock (b_{33}) and the contemporaneous response of inflation to that shock (b_{43}) are allowed to differ across regimes. All other contemporaneous relationships are held fixed. This restriction improves interpretability and helps ensure that regimes are identified by stable and economically meaningful differences in the conditional distribution of innovations.

It is worth distinguishing the present framework from a heteroskedastic SVAR, in which regime dependence is restricted to the error covariance matrix while structural parameters remain constant. Such a model would generate regime-varying second moments but could not separately identify how b_{33} and b_{43} scale across regimes. As a result, it could not decompose the ERPT ratio — defined as b_{43}/b_{33} — into its numerator and denominator components across states, nor determine whether regime differences in measured pass-through arise from shifts in pricing behavior or from variation in exchange rate dynamics. The empirical analysis in Section 3 exploits precisely this decomposition, finding that the dominant source of regime-dependent ERPT is variation in the exchange rate response rather than in the inflation response. This result is only recoverable within a model that separately identifies both structural parameters across regimes. Furthermore, the endogenous feedback from structural shocks to regime probabilities — captured by the vector ρ — provides information about regime dynamics that is entirely absent from variance-switching specifications. The estimated $\hat{\rho}$ implies that inflation surprises systematically alter the likelihood of transitioning between volatility states, a finding with direct policy relevance that cannot be obtained from a model with exogenous transition probabilities.

A further implication is that regime dependence need not translate into regime-dependent pass-through elasticities. Impact pass-through is given by the ratio b_{43}/b_{33}. If b_{33} and b_{43} vary proportionally across regimes, impulse responses differ in magnitude while the implied pass-through elasticity remains approximately invariant. In that case, regime dependence is naturally interpreted as reflecting differences in shock intensity or exchange rate volatility rather than shifts in the structural sensitivity of prices to exchange rate movements. Accordingly, we label the regimes as low- and high-exchange-rate-volatility regimes, and evaluate whether pass-through elasticities vary once responses are scaled by the exchange rate movement induced by the identified shock.

A potential concern is that under Gaussianity the likelihood is primarily informative about changes in $\Sigma(s_t)$ rather than in $B(s_t)$ separately, so that the individual structural parameters b_{33} and b_{43} may be weakly identified even when their ratio is not. Were this the case, the joint posterior of (b_{33}, b_{43}) would exhibit a ridge along a ray through the origin — the signature of a ratio-identified-but-scale-unidentified problem. The empirical relevance of this concern is assessed in Section **??**, where posterior draws confirm that the individual parameters are separately identified.

3 Empirical Illustration: Exchange Rate Pass-Through in Costa Rica

To illustrate the empirical relevance of the proposed endogenous regime-switching SVAR framework, we study exchange rate pass-through in Costa Rica, a small open economy in which exchange rate fluctuations play a central role in inflation dynamics. ERPT refers to the response of domestic prices to movements in the nominal exchange rate.

Costa Rica provides a natural environment to examine state-dependent transmission mechanisms. The USD–CRC exchange rate exhibits periods of marked volatility, while price adjustment is subject to nominal rigidities and institutional frictions. These features suggest that exchange rate shocks may propagate differently across volatility states, even if the long-run elasticity of pass-through remains stable.

Rather than imposing exogenous regime classifications, we allow regime dynamics to emerge endogenously from the interaction between structural shocks and latent state dynamics. In this setting, structural innovations affecting inflation or external conditions may alter the probability of transitioning between low- and high-volatility regimes. Costa Rica therefore offers a suitable empirical laboratory to study whether exchange rate transmission varies systematically across endogenous states of the economy.

3.1 Determining the Baseline Model

The empirical illustration adopts a parsimonious four-variable system designed to isolate exchange rate innovations while controlling for external price pressures. The baseline specification includes: (i) global oil price growth ($\Delta \log WTI$), (ii) U.S. inflation measured by the personal consumption expenditures index ($\Delta \log PCE$), (iii) nominal exchange rate depreciation ($\Delta \log TCN$), and (iv) Costa Rican CPI inflation ($\Delta \log IPC$).

All variables enter in monthly log differences to ensure stationarity and comparability of dynamic responses. Expressing the system in growth rates avoids potential complications arising from mixed integration orders and facilitates interpretation of impulse responses in percentage terms.

Global oil prices capture commodity cost shocks that directly affect domestic production costs and indirectly influence exchange rate dynamics. U.S. inflation reflects broader external price pressures and serves as a proxy for imported inflation beyond energy markets. Together, these variables control for external cost conditions that could otherwise confound the identification of exchange rate innovations.

The recursive ordering places external price variables first, followed by the exchange rate and domestic inflation. This structure treats global price shocks as contemporaneously exogenous to Costa Rica, allows the exchange rate to respond within the month to external disturbances, and permits domestic inflation to respond contemporaneously to all shocks. The ordering reflects the small open economy structure of Costa Rica.

Lag length is selected based on out-of-sample forecasting performance for inflation and the exchange rate across multiple horizons and rolling sample splits. A two-lag specification minimizes forecast errors while maintaining parsimony and is therefore adopted as the baseline. Alternative information sets, recursive orderings, lag lengths, and sample windows are examined in Section 4.

The estimation sample spans January 2009 to September 2025. The starting date is chosen to exclude the period surrounding the global financial crisis, which marked a structural shift in Costa Rica's inflation dynamics and monetary policy framework. Beginning the sample after this episode ensures a more homogeneous inflation environment characterized by inflation targeting and greater exchange rate flexibility, thereby improving the stability and interpretability of the estimated transmission mechanisms.

3.2 Identification, Estimation, and Simulation-Based Inference

Identification combines recursive structural restrictions with regime-dependent impact coefficients. Structural shocks are identified through a lower-triangular contemporaneous impact matrix, while state dependence arises from allowing selected structural parameters to vary across regimes together with endogenous regime transitions governed by the latent state variable.

Estimation. The model is estimated by maximum likelihood. Conditional on the parameter vector, the filtering recursion derived in Section 2 delivers the log-likelihood by integrating over the latent regime process.

Because endogenous regime switching introduces threshold nonlinearities and feedback from structural shocks to regime dynamics, the likelihood surface may exhibit flat regions and multiple local maxima. To mitigate sensitivity to starting values, we implement a multi-start optimization routine. Specifically, the likelihood is maximized repeatedly from dispersed initial conditions, and the parameter vector attaining the highest log-likelihood value is retained as the global optimum.

Simulation-based inference. Uncertainty is quantified using a Gaussian quasi-posterior approximation centered at the maximum likelihood estimate. Let $\hat{\theta}$ denote the full vector of estimated parameters and let $\widehat{V}$ denote the covariance matrix obtained from a numerical approximation to the Hessian of the log-likelihood evaluated at $\hat{\theta}$. We draw

$$\theta^{(b)} \sim \mathcal{N}(\hat{\theta}, \widehat{V}), \qquad b = 1, \ldots, B,$$

and for each draw compute regime-specific impulse responses implied by the corresponding structural matrices.

When reporting regime-dependent impulse responses[3], we condition on the regime remaining fixed over the horizon. This isolates differences in transmission mechanisms across regimes and avoids conflating impulse dynamics with endogenous switching along the response path.

Point estimates are reported as the responses evaluated at $\hat{\theta}$. Uncertainty bands are constructed from the empirical distribution of simulated responses:

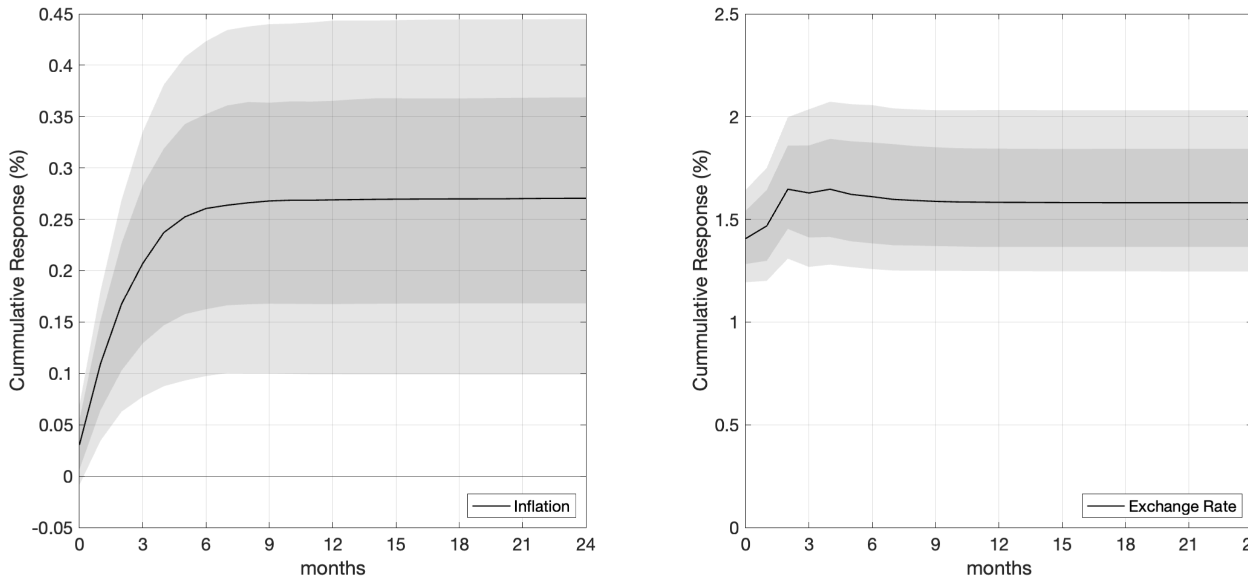
$$\widehat{\mathrm{IRF}}_h = \mathrm{median}\{\mathrm{IRF}_h(\theta^{(b)})\}_{b=1}^{B}, \qquad \mathrm{Band}_h^{(q)} = \mathrm{quantile}_q\{\mathrm{IRF}_h(\theta^{(b)})\}_{b=1}^{B}.$$

Unless otherwise stated, we report 68% and 90% bands corresponding to $q \in \{0.16, 0.84\}$ and $q \in \{0.05, 0.95\}$, respectively.

This procedure propagates parameter uncertainty through both structural transmission and endogenous regime dynamics while remaining computationally tractable in the presence of nonlinear likelihood features.

[3]For the linear SVAR benchmark, we report bootstrap confidence bands; for the endogenous regime-switching model, we use the simulation-based procedure described here.

Figure 2: Linear SVAR impulse responses to an exchange rate shock.



Note: The left panel reports the cumulative CPI response obtained by summing monthly CPI percent changes. The right panel reports the cumulative exchange rate response obtained by summing monthly percent changes in the exchange rate. Shaded areas denote 68% (darker) and 90% (lighter) confidence bands constructed using a residual bootstrap.

3.3 Results from the Linear Model

We begin with a linear SVAR benchmark to assess the extent to which a constant-transmission specification can account for exchange rate pass-through dynamics.

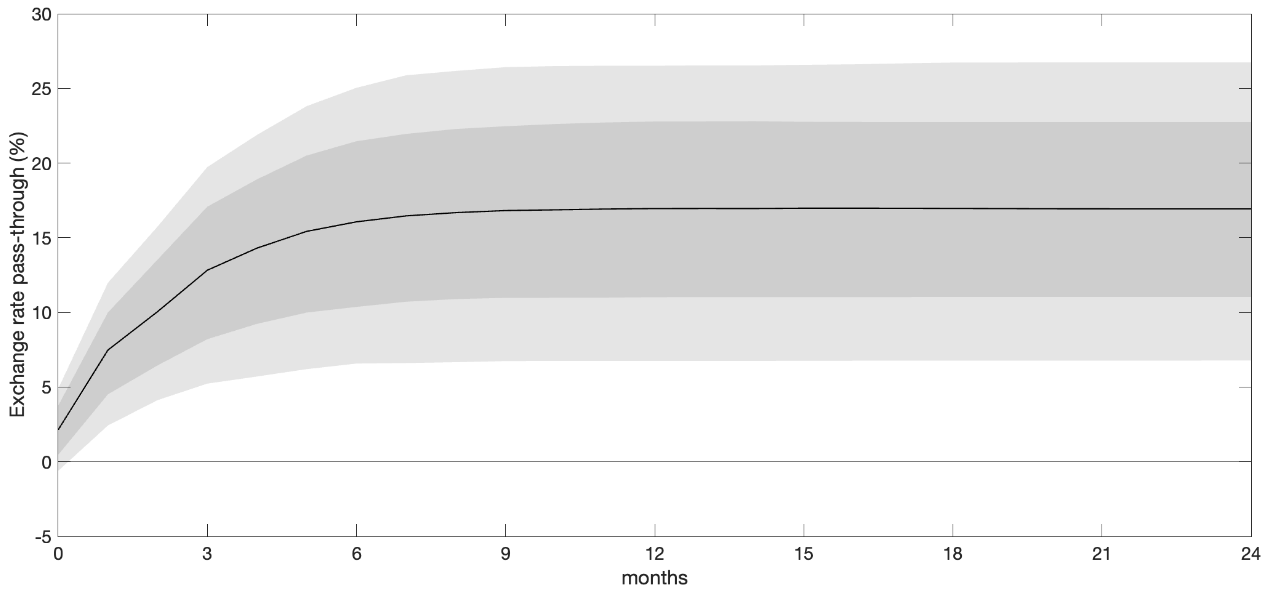
Figure 2 shows that a depreciation generates a gradual but economically meaningful increase in consumer prices. Most of the price adjustment occurs within the first few months, while the exchange rate response is more persistent. These dynamics motivate a cumulative measure of pass-through.

Measuring exchange rate pass-through. Exchange rate pass-through at horizon h is defined as

$$\mathrm{ERPT}_h = 100 \times \frac{\sum_{j=0}^{h} \mathrm{IRF}_{\Delta p,j}}{\sum_{j=0}^{h} \mathrm{IRF}_{\Delta e,j}},$$

where Δp_t and Δe_t denote monthly percent changes in CPI and the exchange rate, respectively. This measure captures the cumulative percent increase in the price level relative to the cumulative percent depreciation of the exchange rate following an identified exchange rate shock.

Figure 3: Cumulative exchange rate pass-through in the linear SVAR.



Note: ERPT is computed as 100 times the cumulative CPI response divided by the cumulative exchange rate response to an exchange rate shock. Shaded areas denote 68% (darker) and 90% (lighter) confidence bands constructed using a residual bootstrap.

Figure 3 reports the implied pass-through elasticity. The linear specification implies moderate and front-loaded transmission. On impact, pass-through is limited, but it rises quickly during the first few months as prices adjust. Cumulative ERPT reaches approximately 9% after one month and 15–16% after three months. The effect peaks around six months at roughly 17% and remains broadly stable thereafter, fluctuating between 16 and 17 percent at one- and two-year horizons.

These results indicate that most exchange rate transmission occurs within the first six months, with limited additional adjustment beyond that horizon. While economically meaningful, the linear model imposes constant transmission dynamics throughout the sample. The next subsection examines whether allowing for endogenous regime switching alters the magnitude and persistence of exchange rate and inflation responses, and whether the implied pass-through elasticity varies across regimes.

3.4 Regime-Dependent Transmission and the Source of Asymmetry

Allowing for endogenous regime switching reveals differences in measured exchange rate pass-through. Importantly, however, the source of this asymmetry does not primarily lie in large differences in the inflation response itself. Instead, it operates mainly through regime-dependent variation in the exchange rate response — that is, through the denominator of the pass-through ratio.

Figure 4 reports cumulative ERPT under the low- and high-volatility regimes together with the linear benchmark. Pass-through is systematically higher in the high-volatility regime (red line) than in the low-volatility regime (blue line), particularly at short and medium horizons.

At horizons of three to six months, cumulative ERPT in the high regime lies several percentage points above that of the low regime. The linear benchmark (black line) typically lies between the two regimes. Although the point estimates indicate stronger pass-through in the high-volatility regime, confidence bands overlap at most horizons, suggesting that the statistical evidence for sharp asymmetry is moderate rather than decisive.

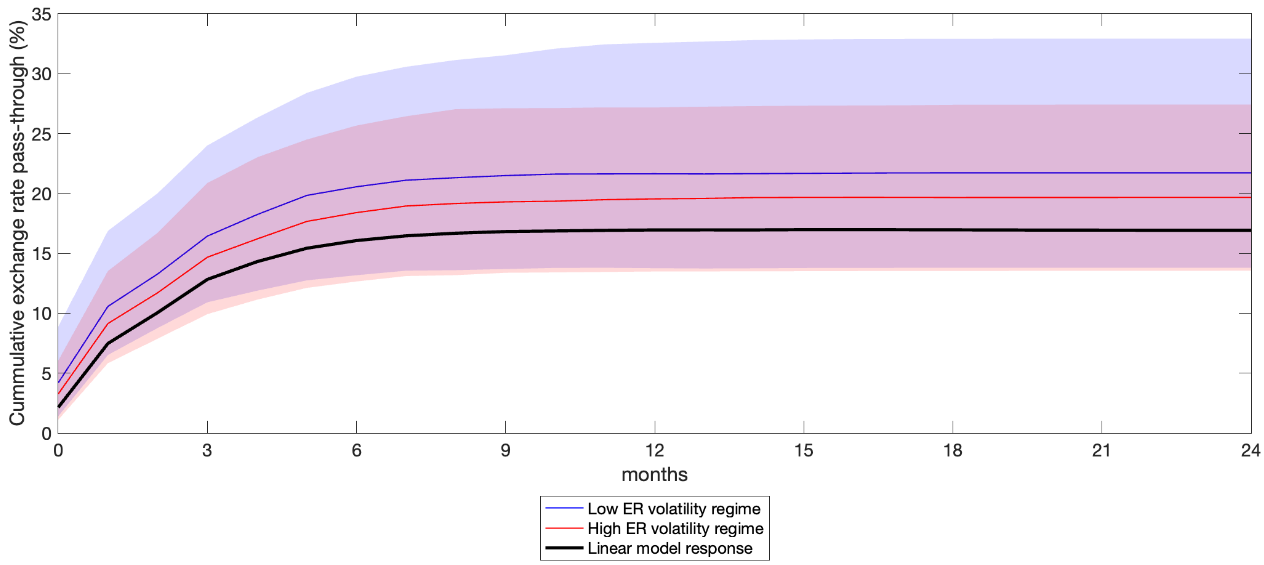
Mechanism: denominator versus numerator variation. To understand the origin of this difference, Figure 5 compares the cumulative impulse responses of the exchange rate and CPI inflation across regimes.

Inflation responses (right panel) are broadly similar in magnitude and persistence across regimes. By contrast, the exchange rate response (left panel) is substantially larger in the low-volatility regime. That is, conditional on an identified exchange rate shock, the exchange rate itself moves more strongly when the system is in the low regime.

Because ERPT is defined as the cumulative CPI response relative to the cumulative exchange rate response, a larger exchange rate response mechanically lowers the ratio, even if the inflation response is similar. The asymmetry in measured pass-through therefore arises mainly from denominator variation (exchange rate dynamics) rather than from large differences in the pricing response of firms.

Distribution of regime differences. Figure 6 presents the posterior distribution of the difference in cumulative ERPT (High minus Low) at selected horizons. The median difference is positive at short and medium horizons, indicating that pass-through is typically stronger in

Figure 4: Regime-dependent cumulative exchange rate pass-through.



Note: Cumulative ERPT for the low-volatility regime (blue), high-volatility regime (red), and the linear SVAR benchmark (black). Shaded areas represent 68% confidence bands obtained from the simulation-based procedure. Point estimates differ modestly across regimes but confidence bands overlap substantially at all horizons, indicating that regime differences in measured pass-through elasticities are not statistically distinguishable. The similar elasticities across regimes, combined with the pronounced differences in exchange rate dynamics documented in Figure 5, suggest that variation in measured pass-through reflects state-dependent exchange rate volatility rather than shifts in pricing behavior.

the high-volatility regime. However, the 68% credible intervals include zero at most horizons.

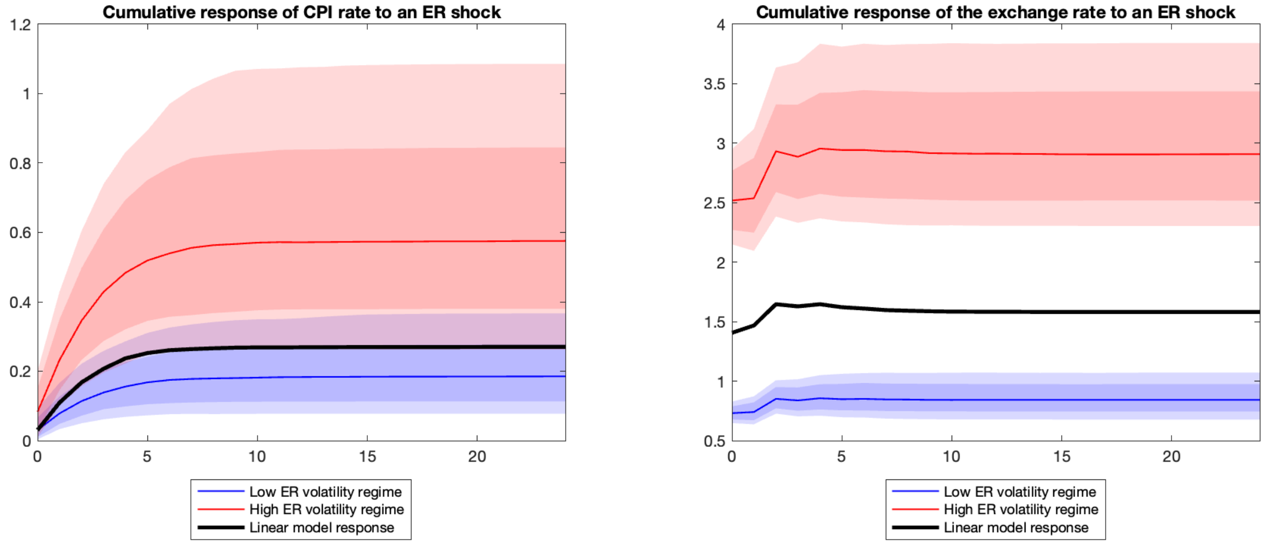
Thus, while point estimates consistently favor higher pass-through in the high-volatility regime, the magnitude of regime asymmetry is economically moderate and not sharply statistically significant at conventional credibility levels.

Impulse responses are computed conditional on the regime remaining fixed over the response horizon. Given the high estimated persistence of the latent state process, this conditioning is empirically reasonable and isolates structural transmission differences across regimes from endogenous switching along the impulse response path.

Overall, the evidence suggests that regime dependence in measured pass-through is driven primarily by state-dependent exchange rate volatility rather than by large shifts in the inflation response itself.

To assess whether this decomposition is supported by the data, Figure 7 reports the marginal posterior distributions of b_{33} and b_{43} across regimes, together with their joint posterior, obtained from simulation-based draws in the neighborhood of the maximum likelihood estimate. The exchange rate own-response b_{33} is sharply separated across regimes with no

Figure 5: Regime-dependent cumulative impulse responses.



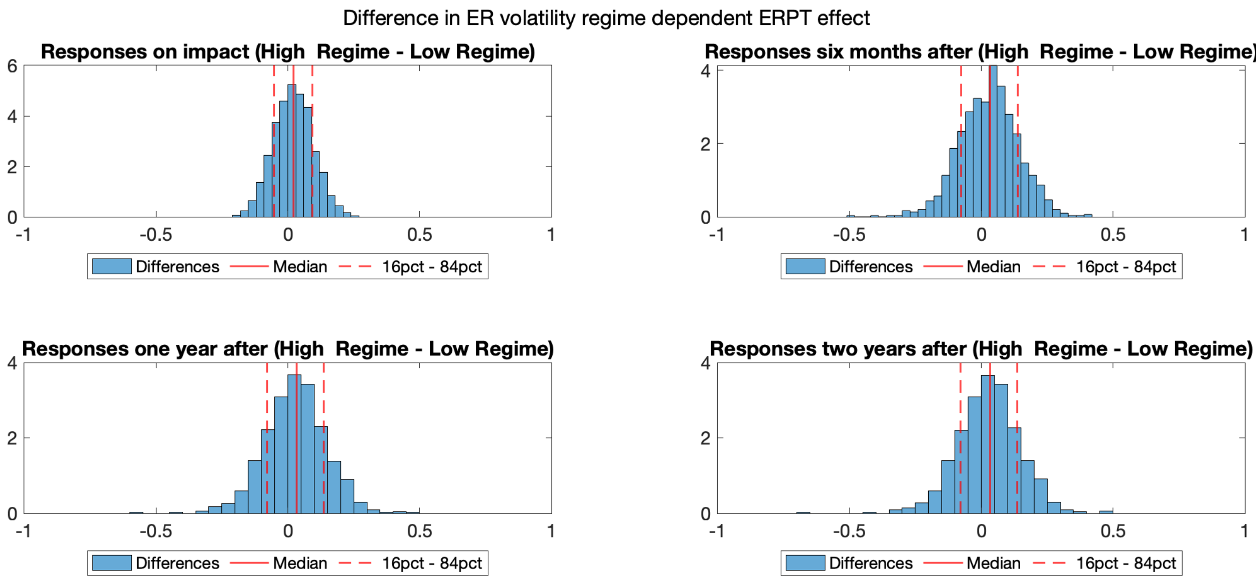
Note: Cumulative responses of CPI inflation (left panel) and the exchange rate (right panel) to an exchange rate shock. Red denotes the high-volatility regime, blue the low-volatility regime, and black the linear benchmark.

distributional overlap, confirming that the data clearly distinguish the magnitude of exchange rate movements across volatility states. The inflation response b_{43} exhibits overlap in the left tails of the two distributions, consistent with the broadly similar inflation impulse responses reported above. Crucially, the joint posterior of (b_{33}, b_{43}) does not exhibit a ridge structure along a ray through the origin — the signature of a ratio-identified-but-scale-unidentified problem flagged in Section 2.6. The vertical spread of the high-volatility cloud, with b_{33} concentrated in a narrow horizontal band while b_{43} varies freely, confirms that the denominator is pinned down independently of the numerator. The individual structural parameters are therefore separately identified, and the decomposition of regime-dependent pass-through into numerator and denominator components is empirically grounded rather than an artifact of the ratio.

3.5 Regime Probabilities and Endogenous Dynamics

Figure 9 displays the filtered estimate of the latent state variable w_t together with the estimated threshold $\hat{\tau} = 1.8751$. Periods in which w_t exceeds the threshold correspond to the high-volatility regime. Figure 10 reports the implied filtered probability of the high-volatility regime alongside the nominal exchange rate. Episodes of pronounced exchange rate fluctu-

Figure 6: Distribution of regime differences in cumulative ERPT (High minus Low)



Note: We present selected horizons. Solid red lines denote medians; dashed lines denote 16th and 84th percentiles.

ations coincide with elevated regime probabilities, while periods of exchange rate stability correspond to low probabilities.

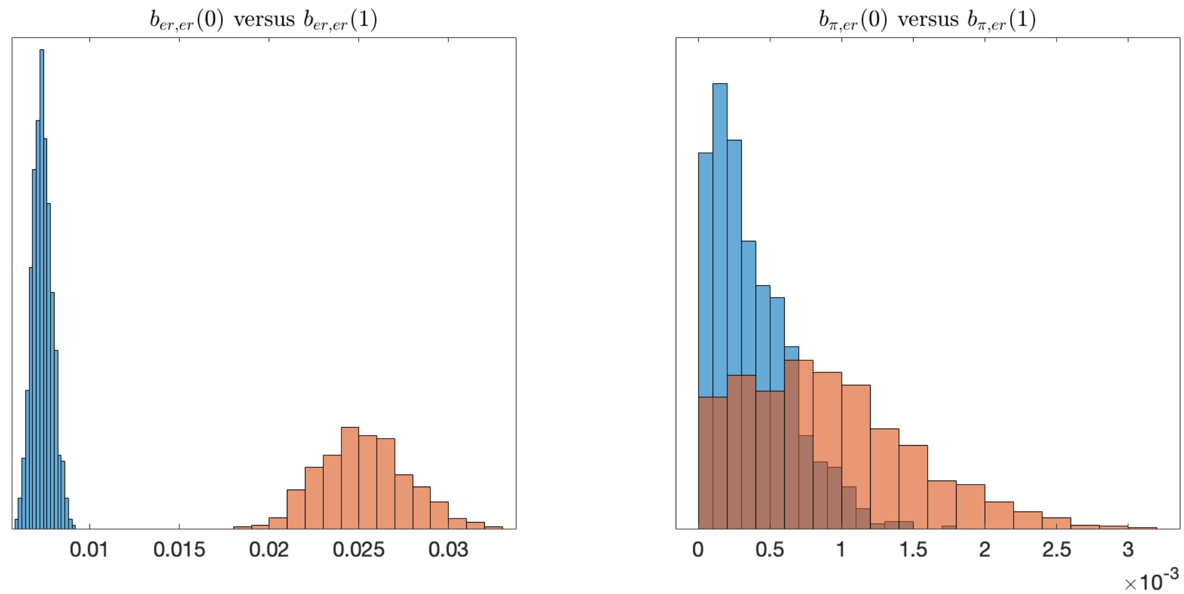
The latent process evolves according to

$$w_t = \alpha w_{t-1} + \rho' \varepsilon_t + u_t,$$

where α governs persistence and ρ captures endogenous feedback from structural shocks to regime dynamics. Two features of the estimated process are worth discussing in turn: its persistence and its responsiveness to structural shocks.

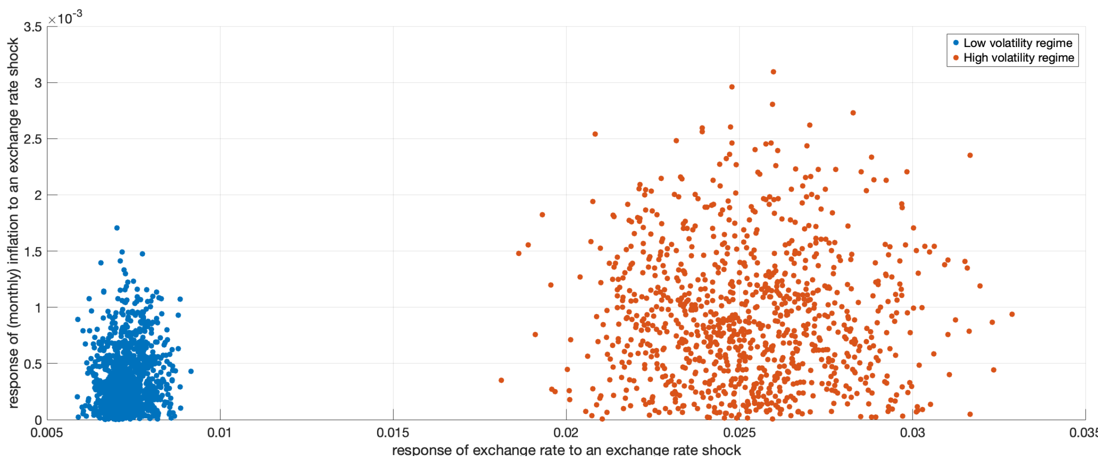
3.5.0.1 Persistence. The estimate $\hat{\alpha} = 0.9491$ implies a highly persistent latent process. With α close to unity, volatility states exhibit substantial duration: once the economy enters a high-volatility regime, it tends to remain there for an extended period. This persistence is reflected in the gradual transitions visible in Figure 10, which arise from a slowly evolving latent component rather than from abrupt, discrete switches.

Figure 7: Quasi-posterior distributions of regime-dependent structural parameters b_{33} and b_{43}



Note: Marginal posterior distributions of the exchange rate own-response b_{33} (left panel) and the contemporaneous inflation response to an exchange rate shock b_{43} (right panel), obtained from simulation-based draws in the neighborhood of the maximum likelihood estimate. Blue denotes the low-volatility regime and orange the high-volatility regime. The complete separation of the b_{33} distributions confirms that the data sharply distinguish exchange rate dynamics across regimes. The overlap in the left tails of the b_{43} distributions is consistent with the broadly similar inflation responses documented in Figure 5. See Figure 8 for the joint posterior.

Figure 8: Joint quasi-posterior of regime-dependent structural parameters (b_{33}, b_{43})



Note: Joint distribution of simulation-based parameter draws for the exchange rate own-response b_{33} (horizontal axis) and the contemporaneous inflation response b_{43} (vertical axis). Blue denotes the low-volatility regime and orange the high-volatility regime. The two clouds are completely separated along the horizontal axis, confirming that b_{33} is sharply identified across regimes. The absence of a ridge structure along a ray through the origin rules out the scenario in which only the ratio b_{43}/b_{33} is identified but not the individual parameters separately. See Figure 7 for the marginal distributions.

3.5.0.2 Endogenous feedback. The estimated feedback vector is

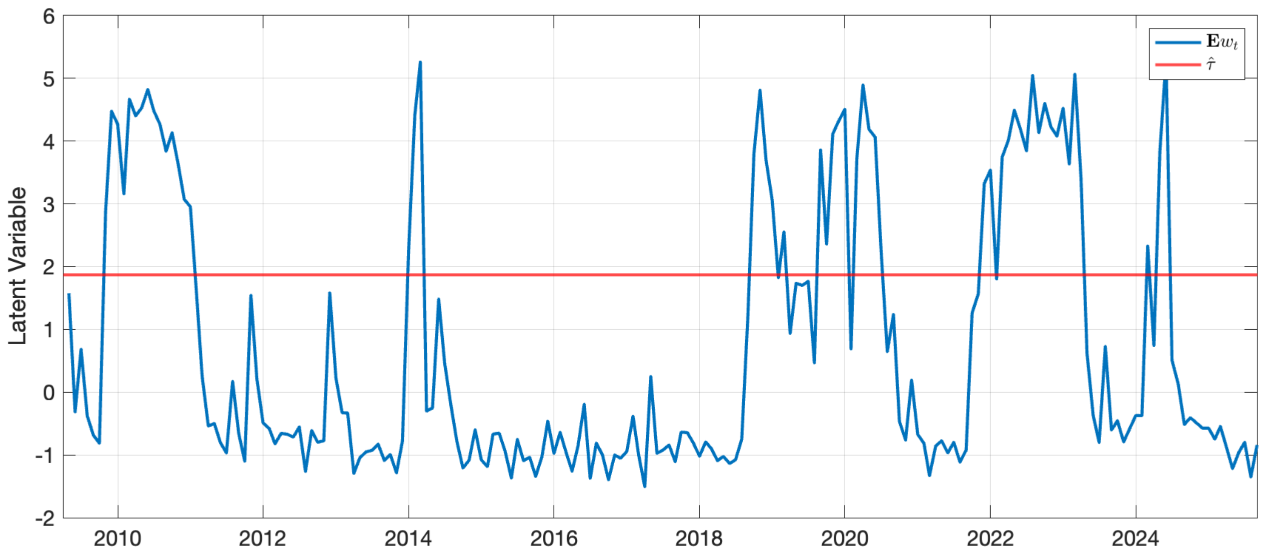
$$\hat{\rho} = \begin{pmatrix} 0.0000 \\ 0.3948 \\ -0.2931 \\ 0.4123 \end{pmatrix}, \qquad \|\hat{\rho}\| = 0.6417,$$

where the elements correspond to oil price, U.S. inflation, exchange rate, and domestic inflation shocks respectively. The overall magnitude $\|\hat{\rho}\| = 0.6417$ indicates economically meaningful endogenous feedback: identified structural shocks alter the probability of crossing the volatility threshold, and regime dynamics are therefore not purely exogenous.

The near-zero loading on oil price shocks suggests that external commodity price disturbances do not directly drive transitions between volatility regimes. This is consistent with the view that global energy price movements affect Costa Rica primarily through their impact on domestic prices rather than through exchange rate volatility per se.

The positive and sizable loading on U.S. inflation shocks ($\hat{\rho}_2 = 0.3948$) indicates that external price pressures increase the probability of entering the high-volatility regime, consistent with the transmission of global inflationary episodes to small open economies through

Figure 9: Estimated expected value of the latent variable w_t and threshold $\hat{\tau}$



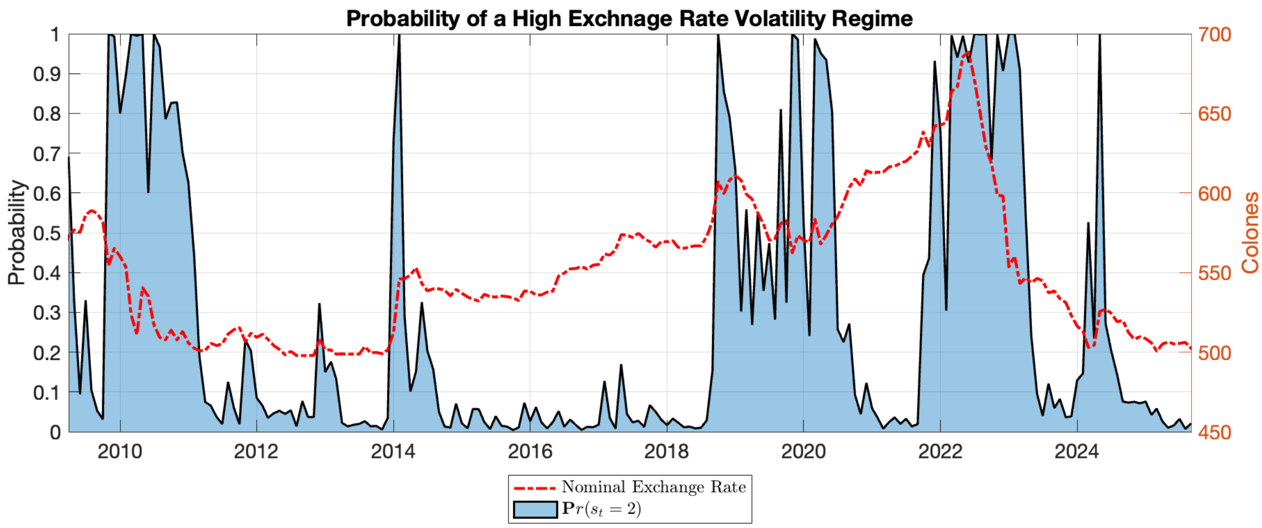
Note: Regimes are determined by whether w_t lies above or below the threshold.

exchange rate pressures.

The negative loading on exchange rate shocks ($\hat{\rho}_3 = -0.2931$) is particularly noteworthy. It implies that an identified exchange rate shock — orthogonal to external conditions and domestic inflation — reduces the probability of transitioning into the high-volatility regime. One interpretation is that idiosyncratic exchange rate movements are associated with subsequent stabilization, either because they reflect mean-reverting dynamics in the bilateral rate or because they trigger a policy response. In Costa Rica, the central bank occasionally intervenes in the foreign exchange market following large exchange rate movements, which could contribute to subsequent volatility compression. While this institutional channel offers a plausible narrative, the monthly frequency of the data does not allow us to separate the identified shock from any within-month policy response, and a more precise assessment is left for future work.

The largest loading corresponds to domestic inflation shocks ($\hat{\rho}_4 = 0.4123$), implying that unexpected domestic price increases raise the probability of elevated exchange rate volatility. This is consistent with an environment in which inflation surprises erode confidence in the nominal anchor, generating subsequent pressure on the exchange rate. Together with the positive loading on U.S. inflation shocks, this finding points to a feedback mechanism in which nominal disturbances — whether external or domestic — systematically push the economy toward the high-volatility regime.

Figure 10: Filtered probability of the high exchange rate volatility regime



Note: For reference the graph also shows the nominal exchange rate in colones (right axis).

Taken together, these estimates indicate that volatility regimes are both persistent and shock-responsive. Regime shifts emerge from the interaction between a highly persistent latent volatility component and contemporaneous structural disturbances, rather than from purely exogenous switching. The formal test of this endogeneity is presented in the following subsection.

3.5.0.3 Endogenous effects. To formally assess whether regime dynamics are endogenous, we conduct a likelihood ratio (LR) test of the null hypothesis that the feedback vector ρ equals zero. Under the null, regime transitions depend solely on the autoregressive component of the latent process and are independent of contemporaneous structural shocks.

We estimate the restricted model by imposing $\rho = 0$ and re-maximizing the likelihood, and compare it to the unrestricted specification in which ρ is freely estimated.

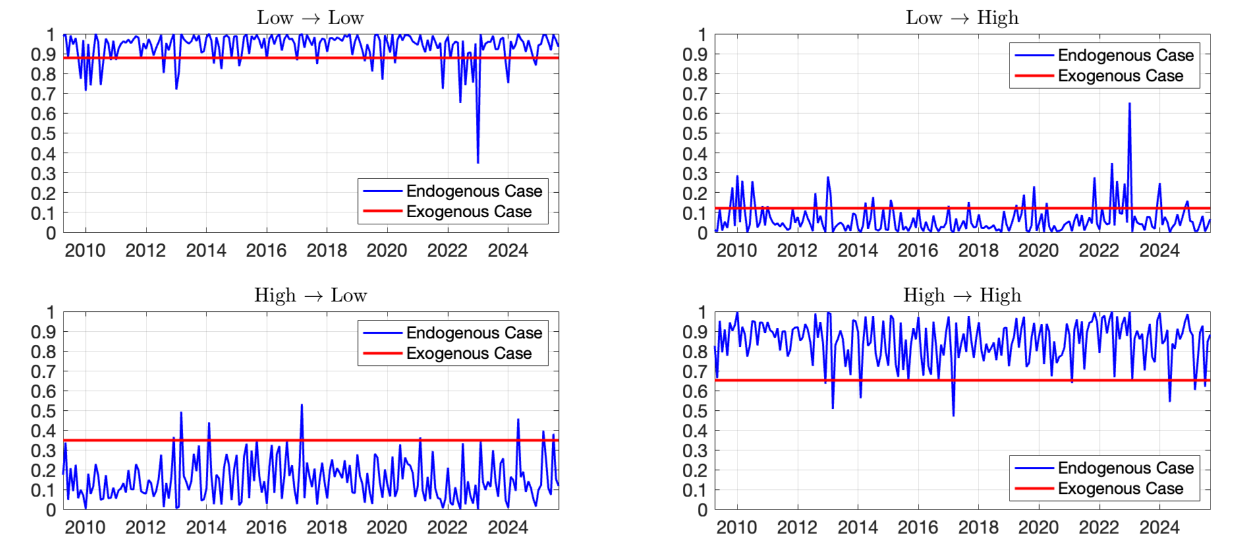
Let ℓ_u and ℓ_r denote the maximized log-likelihoods of the unrestricted and restricted models, respectively. The LR statistic is computed as

$$LR = 2(\ell_u - \ell_r).$$

In our application, $\ell_u = 2607.1$ and $\ell_r = 2601.5$, yielding

$$LR = 2(2607.1 - 2601.5) = 11.2.$$

Figure 11: Time-Varying Transition Probabilities: Endogenous vs. Exogenous Regime Dynamics



Note: Blue lines show estimated transition probabilities under the endogenous specification, in which regime switching depends on structural shocks through the latent process. Red lines show the corresponding constant transition probabilities implied by the restricted exogenous model ($\rho = 0$). The endogenous specification generates substantial time variation in transition probabilities, particularly during episodes of elevated exchange rate volatility.

Under the null hypothesis, the LR statistic is asymptotically distributed as a chi-squared random variable with degrees of freedom equal to the number of restrictions. Since ρ contains four elements, the test involves $q = 4$ restrictions. The 5% critical value for $\chi^2(4)$ is 9.49. Because $11.2 > 9.49$, the null of exogenous regime dynamics is rejected at the 5% significance level (p-value ≈ 0.024).

This result provides statistical evidence that structural shocks influence the evolution of the latent volatility state. Regime transitions are therefore partially endogenous rather than purely exogenous, supporting the mechanism embedded in the model.

4 Robustness Checks

This section evaluates the robustness of the main findings along three dimensions: (i) alternative information sets, (ii) alternative recursive orderings for identification, and (iii) alternative sample periods and data constructions.

Across specifications, the qualitative conclusions remain unchanged. In particular, (i) regime-dependent pass-through persists, (ii) pass-through in the low-volatility regime remains modest, and (iii) differences across regimes continue to be driven primarily by variation in exchange rate dynamics rather than large shifts in the inflation response.

4.1 Description of Alternative Specifications

We assess robustness along three dimensions: (i) the information set included in the VAR, (ii) the recursive ordering used for identification, and (iii) the sample period. Identification is based on a recursive (Cholesky) structure in which variables listed earlier in the system are treated as contemporaneously more exogenous. Unless otherwise noted, the baseline and most alternatives are estimated as a two-regime model; linear (single-regime) counterparts are reported as a benchmark.

Benchmark specification. The baseline model includes four variables in monthly growth rates: global oil prices ($\Delta \log WTI$), U.S. inflation measured by the personal consumption expenditures index ($\Delta \log PCE$), the nominal exchange rate depreciation ($\Delta \log TCN$), and Costa Rican CPI inflation ($\Delta \log IPC$). This ordering treats external price pressures as predetermined relative to domestic variables and allows exchange rate movements to affect inflation contemporaneously within the month.

Expanded system with external monetary conditions and domestic activity. We enlarge the information set to include U.S. monetary conditions and Costa Rican domestic fundamentals: $\Delta \log WTI$, $\Delta \log PCE$, a ZLB-adjusted U.S. policy-rate measure (the shadow federal funds rate), domestic economic activity growth ($\Delta \log IMAE$), exchange rate depreciation ($\Delta \log TCN$), the domestic policy rate (TPM, in percentage points), and CPI inflation ($\Delta \log IPC$). Global prices and U.S. monetary conditions are ordered first, reflecting their exogeneity to Costa Rica. Domestic activity, the exchange rate, and monetary policy respond contemporaneously to external conditions, while inflation is ordered last. We report the linear (single-regime) version of this specification.

Parsimonious external-controls specification. To verify that the results are not driven by the richer domestic block, we estimate a parsimonious four-variable model including $\Delta \log WTI$, the shadow federal funds rate, $\Delta \log TCN$, and $\Delta \log IPC$. This specification retains external cost and monetary conditions but excludes domestic activity and the domestic policy rate.

Alternative domestic ordering within the expanded system. Within the expanded information set, we also consider an alternative recursive ordering in which the exchange rate precedes domestic economic activity. This allows exchange rate innovations to affect activity contemporaneously within the month and provides a robustness check on the timing assumptions embedded in the benchmark ordering (we again report the linear version).

Alternative domestic ordering in a Costa Rica–specific system. As an additional robustness check that uses Costa Rica–constructed rates directly (rather than log changes), we estimate a four-variable two-regime model ordered as follows: the Central Bank of Costa Rica oil-price index rate, trading-partner inflation, the monthly percent change in the nominal exchange rate, and monthly Costa Rican inflation. This specification mirrors the benchmark timing assumptions while relying on domestic statistical constructions.

Restricted sample. Finally, we assess sensitivity to sample composition by re-estimating the four-variable system on a shorter, fixed subsample. The variable set and ordering remain unchanged; only the estimation window differs.

Taken together, these alternatives evaluate whether the regime-dependent ERPT results depend on the choice of controls, the assumed timing structure, the use of rates versus log changes, or the sample period.

Table 1 reports cumulative exchange rate pass-through at selected horizons for the benchmark model and the alternative specifications. For expositional clarity, we report point estimates only; confidence bands for all alternatives are available upon request.

Table 1: Regime-Dependent Responses Across Alternative Specifications

	0M			3M			6M			12M			24M		
Model	High	Low	Linear	High	Low	Linear	High	Low	Linear	High	Low	Linear	High	Low	Linear
Panel A: Inflation Response to ER Shock															
Benchmark	0.0824 [0.0282,0.1501]	0.0306 [0.0103,0.0658]	0.0303 [0.0303,0.0303]	0.4285 [0.2869,0.6088]	0.1385 [0.0905,0.2053]	0.2070 [0.2070,0.2070]	0.5396 [0.3570,0.7877]	0.1748 [0.1080,0.2539]	0.2605 [0.2605,0.2605]	0.5716 [0.3784,0.8390]	0.1832 [0.1126,0.2745]	0.2689 [0.2689,0.2689]	0.5756 [0.3799,0.8450]	0.1851 [0.1130,0.2771]	0.2704 [0.2704,0.2704]
Alt 1: Expanded Information Set (7-var, 1 lag)	0.0919 [0.0342,0.1614]	0.0404 [0.0128,0.0798]	0.0317 [0.0317,0.0317]	0.2967 [0.1186,0.4980]	0.1042 [0.0452,0.1801]	0.1305 [0.1305,0.1305]	0.3051 [0.0957,0.5469]	0.1068 [0.0382,0.1930]	0.1328 [0.1328,0.1328]	0.3154 [0.0951,0.5601]	0.1098 [0.0372,0.1970]	0.1389 [0.1389,0.1389]	0.3341 [0.0915,0.5914]	0.1132 [0.0302,0.2065]	0.1491 [0.1491,0.1491]
Alt 2: Extended information set (7 variables)	0.0900 [0.0339,0.1586]	0.0304 [0.0092,0.0609]	0.0222 [0.0222,0.0222]	0.4185 [0.2310,0.5989]	0.1236 [0.0728,0.1805]	0.1605 [0.1605,0.1605]	0.4934 [0.2798,0.7127]	0.1429 [0.0857,0.2046]	0.1871 [0.1871,0.1871]	0.5016 [0.3045,0.7366]	0.1468 [0.0910,0.2109]	0.1948 [0.1948,0.1948]	0.5248 [0.3341,0.7764]	0.1516 [0.0985,0.2221]	0.2037 [0.2037,0.2037]
Alt 3: CR Rate-Based Specification	0.1246 [0.0477,0.2159]	0.0324 [0.0097,0.0677]	0.0380 [0.0380,0.0380]	0.5612 [0.3805,0.7993]	0.1855 [0.1197,0.2594]	0.2170 [0.2170,0.2170]	0.6702 [0.4308,0.9822]	0.2209 [0.1366,0.3246]	0.2391 [0.2391,0.2391]	0.6755 [0.4261,0.9947]	0.2209 [0.1344,0.3312]	0.2364 [0.2364,0.2364]	0.6472 [0.4086,0.9699]	0.2130 [0.1217,0.3274]	0.2263 [0.2263,0.2263]
Alt 4: Pre-Pandemic Sample	0.1150 [0.0411,0.2069]	0.0441 [0.0124,0.0892]	0.0258 [0.0258,0.0258]	0.2463 [0.1001,0.4219]	0.0870 [0.0329,0.1591]	0.0867 [0.0867,0.0867]	0.2643 [0.1029,0.4713]	0.0922 [0.0366,0.1720]	0.0899 [0.0899,0.0899]	0.2627 [0.0980,0.4801]	0.0926 [0.0363,0.1760]	0.0880 [0.0880,0.0880]	0.2477 [0.0839,0.4720]	0.0885 [0.0337,0.1742]	0.0807 [0.0807,0.0807]
Alt 5: Expanded System – Alternative Ordering (1 lag)	0.1104 [0.0427,0.1957]	0.0547 [0.0173,0.1042]	0.0365 [0.0365,0.0365]	0.3274 [0.1241,0.5694]	0.1273 [0.0333,0.2336]	0.1367 [0.1367,0.1367]	0.3327 [0.1013,0.6296]	0.1249 [0.0278,0.2495]	0.1399 [0.1399,0.1399]	0.3484 [0.1091,0.6418]	0.1270 [0.0274,0.2518]	0.1465 [0.1465,0.1465]	0.3629 [0.1034,0.6608]	0.1333 [0.0241,0.2707]	0.1559 [0.1559,0.1559]
Panel B: Exchange Rate Response															
Benchmark	2.5177 [2.2719,2.7682]	0.7309 [0.6783,0.7875]	1.4061 [1.4061,1.4061]	2.8859 [2.5309,3.3237]	0.8377 [0.7515,0.9468]	1.6284 [1.6284,1.6284]	2.9428 [2.5429,3.4464]	0.8507 [0.7540,0.9850]	1.6102 [1.6102,1.6102]	2.9122 [2.5181,3.4303]	0.8426 [0.7451,0.9753]	1.5825 [1.5825,1.5825]	2.9081 [2.5185,3.4362]	0.8430 [0.7451,0.9748]	1.5808 [1.5808,1.5808]
Alt 1: Expanded Information Set (7-var, 1 lag)	2.4294 [2.1868,2.6987]	0.6447 [0.5948,0.7040]	1.3647 [1.3647,1.3647]	2.3695 [2.0281,2.7407]	0.6354 [0.5451,0.7357]	1.3339 [1.3339,1.3339]	2.3687 [1.9877,2.7819]	0.6384 [0.5298,0.7600]	1.3504 [1.3504,1.3504]	2.3887 [1.9314,2.8424]	0.6450 [0.5078,0.7770]	1.3711 [1.3711,1.3711]	2.3832 [1.8718,2.9235]	0.6469 [0.4797,0.8108]	1.3968 [1.3968,1.3968]
Alt 2: External Monetary Controls (Oil + FFR, 4-var)	2.6851 [2.3959,3.0098]	0.7190 [0.6677,0.7825]	1.3873 [1.3873,1.3873]	3.0978 [2.6310,3.5968]	0.8217 [0.7208,0.9488]	1.5066 [1.5066,1.5066]	3.0976 [2.6205,3.6701]	0.8244 [0.7093,0.9700]	1.4918 [1.4918,1.4918]	3.0961 [2.6030,3.6727]	0.8222 [0.7054,0.9709]	1.4846 [1.4846,1.4846]	3.1132 [2.6041,3.7375]	0.8274 [0.6993,0.9816]	1.4915 [1.4915,1.4915]
Alt 3: CR Rate-Based Specification	2.8794 [2.3477,3.4617]	1.0125 [0.9133,1.1140]	1.3966 [1.3966,1.3966]	3.2303 [2.5524,4.0884]	1.1418 [0.9561,1.3768]	1.4952 [1.4952,1.4952]	3.1884 [2.4704,4.1184]	1.1310 [0.9186,1.3929]	1.4379 [1.4379,1.4379]	3.0518 [2.3293,4.0430]	1.0925 [0.8497,1.3622]	1.4012 [1.4012,1.4012]	2.9513 [2.1307,3.9925]	1.0507 [0.8015,1.3511]	1.3654 [1.3654,1.3654]
Alt 4: Pre-Pandemic Sample	2.2902 [1.9823,2.5989]	0.6705 [0.6050,0.7476]	1.3146 [1.3146,1.3146]	2.2401 [1.8179,2.7134]	0.6520 [0.5336,0.8122]	1.1714 [1.1714,1.1714]	2.2698 [1.8723,2.7458]	0.6563 [0.5395,0.8264]	1.2010 [1.2010,1.2010]	2.2702 [1.8728,2.7613]	0.6596 [0.5411,0.8283]	1.2026 [1.2026,1.2026]	2.2872 [1.8786,2.7801]	0.6660 [0.5432,0.8346]	1.2084 [1.2084,1.2084]
Alt 5: Expanded System – Alternative Ordering (1 lag)	2.4926 [2.1641,2.8772]	0.6628 [0.5859,0.7486]	1.3749 [1.3749,1.3749]	2.3713 [1.9502,2.7982]	0.5803 [0.4154,0.7083]	1.3303 [1.3303,1.3303]	2.3737 [1.9293,2.8673]	0.5916 [0.4013,0.7363]	1.3446 [1.3446,1.3446]	2.3838 [1.8740,2.9124]	0.5902 [0.3661,0.7751]	1.3692 [1.3692,1.3692]	2.3822 [1.8043,2.9885]	0.5921 [0.3083,0.8321]	1.3989 [1.3989,1.3989]
Panel C: Implied Pass-Through (Percent)															
Benchmark	3.2626 [1.1086,6.0436]	4.1912 [1.4008,8.8765]	2.1418 [0.4700,3.7527]	14.6755 [9.9260,20.8633]	16.4390 [10.9183,24.0011]	12.8250 [8.2096,17.0766]	18.4015 [12.6538,25.6636]	20.5583 [13.1777,29.7455]	16.0654 [10.3696,21.4608]	19.5474 [13.4879,27.1746]	21.6422 [13.7645,32.5712]	16.9534 [11.0198,22.7828]	19.6682 [13.5556,27.4225]	21.7213 [13.8011,32.9178]	16.9302 [11.0335,22.7420]
Alt 1: Expanded Information Set (7-var, 1 lag)	3.7422 [1.4094,6.8316]	6.2633 [1.9403,12.1811]	2.3129 [0.6334,4.1813]	12.5659 [5.0741,21.4880]	16.5008 [7.0545,28.2109]	9.6656 [5.4545,13.8227]	12.8456 [4.1513,23.2674]	17.0816 [5.9959,30.9121]	9.7659 [5.6204,14.2570]	13.2525 [4.1477,24.2592]	17.3589 [5.6153,31.8297]	10.0909 [6.0892,14.8047]	13.8170 [4.0591,24.7103]	17.5704 [4.9384,32.9568]	10.6994 [6.3749,15.9207]
Alt 2: External Monetary Controls (Oil + FFR, 4-var)	3.4061 [1.2193,5.8718]	4.3085 [1.2624,8.3271]	1.6267 [0.1704,3.2529]	13.0766 [7.7209,19.3826]	14.8518 [8.5787,22.3219]	10.6505 [6.2686,14.9997]	15.5275 [8.7617,22.6149]	17.2732 [10.0396,25.5092]	12.5507 [7.6447,17.9532]	16.2426 [9.7676,23.4841]	17.7865 [11.0714,25.8090]	13.1644 [8.2268,18.3343]	16.8717 [10.9454,24.3040]	18.3803 [11.7796,26.8196]	13.8514 [8.5223,19.0926]
Alt 3: CR Rate-Based Specification	4.2773 [1.6841,7.6210]	3.2261 [0.9710,6.7430]	2.6914 [1.0823,4.3620]	17.2539 [12.1843,23.5375]	16.0637 [10.7590,21.9648]	14.5209 [10.2944,18.7251]	21.0223 [14.3140,28.9665]	19.5164 [12.7430,27.1441]	16.7366 [11.4765,22.1565]	21.9904 [14.2886,30.8378]	20.4068 [12.3581,29.5070]	16.7924 [11.2616,22.7719]	21.8719 [13.8172,32.0843]	20.3050 [12.0118,31.1672]	16.4746 [10.8504,23.2544]
Alt 4: Pre-Pandemic Sample	5.0063 [1.7781,9.3150]	6.4086 [1.8603,13.0310]	1.9534 [−0.0549,3.8201]	11.0966 [4.3846,18.6589]	13.0276 [5.0036,25.0484]	7.5059 [2.1596,12.9089]	11.6216 [4.3900,20.5262]	13.8398 [5.3308,26.2667]	7.4815 [2.3486,12.4720]	11.4637 [4.2852,20.6154]	13.8245 [5.4135,26.0972]	7.2353 [2.1230,12.3365]	10.8997 [3.6325,20.1176]	13.3462 [5.0369,25.4718]	6.7573 [1.6865,11.8512]
Alt 5: Expanded System – Alternative Ordering (1 lag)	4.4600 [1.7698,7.9059]	8.3665 [2.4817,15.8889]	2.6329 [1.0074,4.4088]	13.9486 [5.5823,24.3321]	21.5187 [5.6680,42.9454]	10.0777 [6.5809,14.3757]	14.0430 [4.4973,26.5682]	21.1579 [3.6303,42.6919]	10.2175 [6.5565,14.8830]	14.2662 [4.8700,26.5433]	20.8454 [2.5702,45.3816]	10.5991 [6.9133,15.4080]	15.3772 [4.6429,28.3905]	21.1956 [1.1168,49.2638]	11.2538 [7.0953,16.1814]

Across all specifications, impact ERPT remains economically modest, generally ranging between approximately 3% and 5% in both regimes. Although point estimates vary across models, the regime-dependent pattern becomes more pronounced at medium and longer horizons. In particular, cumulative pass-through at 12 to 24 months is substantially larger in the high-volatility regime than in the low-volatility regime across most specifications. While magnitudes differ moderately, the qualitative evidence of regime dependence remains intact.

4.2 Pre-Pandemic Sample

To assess sensitivity to the pandemic period, we re-estimate the benchmark specification using a sample ending in December 2019. The resulting regime-dependent ERPT remains broadly consistent with the full-sample results. In the pre-pandemic subsample, impact ERPT equals 5.01% in the high-volatility regime and 4.09% in the low-volatility regime.

Although point estimates differ modestly from the full-sample results, they remain within the confidence bands of the benchmark specification. Moreover, the divergence between regimes becomes more pronounced at longer horizons, consistent with the full-sample evidence. The regime classification is broadly stable, with high-volatility episodes coinciding with periods of pronounced exchange rate fluctuations. These findings indicate that the main conclusions are not driven by the inclusion of the pandemic period.

5 Broader Implications of the Framework

Institutional pricing and the absence of sign asymmetry. An institutional explanation for the limited evidence of pronounced sign asymmetry may lie in the regulated pricing of fuel in Costa Rica. Fuel prices are set administratively by ARESEP using a cost-based formula that incorporates international fuel prices and the exchange rate. Because import costs are denominated in U.S. dollars and converted mechanically into colones, exchange rate appreciations and depreciations enter the pricing formula symmetrically.

Given the relatively large weight of fuel in the CPI basket and its salience for inflation expectations, this institutional structure may contribute to the broadly similar inflation responses observed across regimes. In this environment, exchange rate movements affect prices through a rule-based and transparent mechanism, potentially dampening discretionary or asymmetric pricing behavior at the retail level. While our analysis does not identify this channel causally, it provides a coherent interpretation consistent with the empirical findings.

5.1 Implications for exchange rate pass-through

This section summarizes the main findings and discusses their broader implications for economic analysis in small open economies.

Pass-through as a state-dependent phenomenon. Our results indicate that treating exchange rate pass-through as a constant parameter can obscure economically meaningful variation over time. The data support the presence of endogenous regimes in which the measured pass-through ratio differs across states. However, this regime dependence does not primarily reflect large structural shifts in the inflation response itself. Rather, it reflects differences in the magnitude of exchange rate movements across regimes.

The dominant mechanism operates through exchange rate dynamics. The central asymmetry in Costa Rica appears to arise through denominator variation in the pass-through ratio. Inflation responses to identified exchange rate shocks are broadly similar across regimes, while the exchange rate response varies substantially in magnitude. Consequently, measured pass-through differs across states even when the pricing response of firms remains relatively stable. This distinction is important for interpretation: variation in pass-through need not imply variation in price-setting behavior.

Endogenous regime dynamics and macroeconomic shocks. Structural shocks influence not only macroeconomic outcomes but also the probability of switching between regimes. In particular, unexpected inflation shocks increase the likelihood of transitioning into the low pass-through regime. This finding suggests a feedback mechanism whereby inflation surprises are associated with subsequent exchange rate volatility, reinforcing regime persistence. Such endogenous dynamics cannot be captured by models with exogenous transition probabilities.

5.2 Broader methodological implications

Beyond the Costa Rican case, the framework illustrates how endogenous regime-switching methods can be embedded within a structural VAR environment to study nonlinear transmission mechanisms in small open economies.

More broadly, the approach is applicable to settings in which structural shocks affect not only macroeconomic outcomes but also the likelihood of transitioning between states.

Examples include state-dependent fiscal multipliers, nonlinear financial stress transmission, monetary regime shifts, and nonlinearities in inflation expectations.

In such environments, allowing regime probabilities to respond endogenously to identified shocks provides a disciplined way to model nonlinear adjustment dynamics without imposing exogenous regime classifications.

6 Conclusion

This paper studies the exchange rate pass-through effect in Costa Rica using a regime-switching structural vector autoregression with endogenous regime dynamics. Rather than treating pass-through as a constant structural parameter, we allow both transmission mechanisms and regime probabilities to evolve endogenously in response to identified structural shocks.

The evidence supports the presence of state-dependent pass-through dynamics. We identify two regimes in which the measured pass-through ratio differs across states. In the low-volatility regime, impact pass-through is modest and short-lived, while in the high-volatility regime the measured pass-through ratio is substantially larger. However, the primary source of this regime dependence lies in differences in exchange rate dynamics rather than in pronounced shifts in the inflation response itself. Inflation responses to identified exchange rate shocks are broadly similar across regimes, whereas the exchange rate response varies considerably in magnitude. As a result, variation in the denominator of the pass-through ratio plays a central role in generating regime-dependent elasticities.

We also document endogenous regime dynamics. Unexpected inflation shocks increase the probability of transitioning into the low pass-through regime, suggesting a feedback mechanism linking inflation surprises to subsequent exchange rate volatility. These findings underscore the importance of allowing structural shocks to influence not only macroeconomic outcomes but also regime probabilities.

Institutional features of the Costa Rican economy, particularly the rule-based and exchange-rate-linked pricing of regulated fuel, provide a plausible interpretation for the absence of strong sign asymmetry in inflation responses. While this channel is not identified causally in our framework, it offers a coherent explanation consistent with the empirical evidence.

More broadly, the results highlight the importance of modelling nonlinear transmission mechanisms in small open economies. Embedding endogenous regime-switching dynamics within a structural VAR environment provides a flexible and disciplined approach to studying state-dependent macroeconomic adjustment without imposing exogenous regime classifications.

Future research may further explore how exchange rate volatility interacts with real economic activity and expectations formation, and whether similar mechanisms operate in other small open economies.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this manuscript, the authors used AI -assisted tools for language refinement and clarity improvement. The authors reviewed and edited all content and take full responsibility for the accuracy and integrity of the manuscript.

References

Brenes, C. and Esquivel, M. (2017). Asimetrías en el traspaso del tipo de cambio durante el periodo de flexibilidad cambiaria en costa rica.

Castrillo, D. and Laverde, B. (2007). Validación y actualización del modelo de pass through del tipo de cambio en costa rica 1991 -2007.

Chang, Y., Choi, Y. and Park, J. Y. (2017). A new approach to model regime switching. *Journal of Econometrics*, **196** (1), 127 – 143.

Esquivel, M. and Gomez-Rodriguez, F. (2010). Asymmetries of the exchange rate pass through to domestic prices: The case of costa rica.

Forbes, K., Hjortsoe, I. and Nenova, T. (2018). The shocks matter: improving our estimates of exchange rate pass-through. *Journal of international economics*, **114**, 255–275.

Guillermo Peón, S. B. and Rodríguez Brindis, M. A. (2014). Analyzing the exchange rate pass-through in mexico: Evidence post inflation targeting implementation. *Ensayos sobre Política Económica*, **32** (74), 18–35.

Ha, J., Stocker, M. M. and Yilmazkuday, H. (2020). Inflation and exchange rate pass-through. *Journal of International Money and Finance*, **105**, 102187.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the econometric society*, pp. 357–384.

Krolzig, H.-M. (1997). The markov-switching vector autoregressive model. In *Markov-Switching Vector Autoregressions*, Springer, pp. 6–28.

León, J., Morera, A. and Ramos, W. (2001). El pass through del tipo de cambio: Un análisis para la economía costarricense de 1991 al 2001.

Rodríguez, A. (2009). Evaluación del modelo lineal de pass-through para la proyección de inflación dentro del régimen de banda cambiaria.

A Proofs

The following results adapt the univariate arguments developed in Chang *et al.* (2017) to the multivariate setting considered in this paper. The structure of the proof closely follows their approach, with modifications required to accommodate vector-valued innovations and multivariate dependence.

Proof of the Theorem

We begin from equation (6), which after standarization of η_t implies

$$\tilde{\eta}_t = \frac{w_t - \alpha w_{t-1} - \rho_m' \varepsilon_{t-1}^m}{\sqrt{1 - \|\rho_m\|^2}}.$$

By construction,

$$\tilde{\eta}_t \mid (w_{t-1}, \varepsilon_{t-1}^m) \sim \mathcal{N}(0,1).$$

Step 1: Conditional transition probability of the latent variable. Using the above normalization, we obtain

$$\begin{aligned}
\mathbb{P}(w_t < \tau \mid w_{t-1}, \varepsilon_{t-1}^m) &= \mathbb{P}\left(\tilde{\eta}_t < \frac{\tau - \alpha w_{t-1} - \rho_m' \varepsilon_{t-1}^m}{\sqrt{1 - \|\rho_m\|^2}} \,\middle|\, w_{t-1}, \varepsilon_{t-1}^m\right) \\
&= \Phi\left(\frac{\tau - \alpha w_{t-1} - \rho_m' \varepsilon_{t-1}^m}{\sqrt{1 - \|\rho_m\|^2}}\right).
\end{aligned}$$

Moreover, since w_t depends on the past only through $(w_{t-1}, \varepsilon_{t-1}^m)$, we have

$$p(w_t \mid w_{t-1}, w_{t-2}, \varepsilon_{t-1}^m, \varepsilon_{t-2}^m) = p(w_t \mid w_{t-1}, \varepsilon_{t-1}^m).$$

In addition, w_{t-1} is independent of ε_{t-1}^m.

Step 2: Transition probabilities for the regime indicator. Recall that $s_t = 0$ if $w_t < \tau$ and $s_t = 1$ otherwise. Conditioning on s_{t-1} yields

$$
\begin{aligned}
\mathbb{P}(s_t = 0 \mid s_{t-1} = 0, \varepsilon^m_{t-1}) &= \mathbb{P}(w_t < \tau \mid w_{t-1} < \tau, \varepsilon^m_{t-1}) \\
&= \frac{\int_{-\infty}^{\tau\sqrt{1-\alpha^2}} \Phi\left(\frac{\tau - \rho_m' \varepsilon^m_{t-1}}{\sqrt{1-\|\rho_m\|^2}} - \frac{\alpha x}{\sqrt{1-\alpha^2}\sqrt{1-\|\rho_m\|^2}}\right) \varphi(x)\, dx}{\Phi(\tau\sqrt{1-\alpha^2})},
\end{aligned}
$$

where we use the fact that $w_{t-1}\sqrt{1-\alpha^2} \overset{d}{=} \mathcal{N}(0,1)$.

Similarly,

$$
\begin{aligned}
\mathbb{P}(s_t = 0 \mid s_{t-1} = 1, \varepsilon^m_{t-1}) &= \mathbb{P}(w_t < \tau \mid w_{t-1} \geq \tau, \varepsilon^m_{t-1}) \\
&= \frac{\int_{\tau\sqrt{1-\alpha^2}}^{\infty} \Phi\left(\frac{\tau - \rho_m' \varepsilon^m_{t-1}}{\sqrt{1-\|\rho_m\|^2}} - \frac{\alpha x}{\sqrt{1-\alpha^2}\sqrt{1-\|\rho_m\|^2}}\right) \varphi(x)\, dx}{1 - \Phi(\tau\sqrt{1-\alpha^2})}.
\end{aligned}
$$

These expressions deliver the regime transition probabilities conditional on lagged structural shocks.

Step 3: First-order Markov property. We can write the joint conditional density as

$$
\begin{aligned}
p(s_t, \varepsilon^m_t \mid s_{t-1}, \ldots, s_1, \varepsilon^m_{t-1}, \ldots, \varepsilon^m_1) &= p(\varepsilon^m_t \mid s_t, s_{t-1}, \varepsilon^m_{t-1}) \\
&\quad \times p(s_t \mid s_{t-1}, \varepsilon^m_{t-1}).
\end{aligned}
$$

Both components depend on the past only through $(s_{t-1}, \varepsilon^m_{t-1})$, implying

$$
p(s_t, \varepsilon^m_t \mid \cdot) = p(s_t, \varepsilon^m_t \mid s_{t-1}, \varepsilon^m_{t-1}).
$$

Hence, (s_t, ε^m_t) forms a first-order Markov process. □

Proof of the Corollary

We consider the case $0 < \alpha < 1$. The cases $\alpha = 0$ and $-1 < \alpha < 0$ follow by straightforward modifications.

By definition,

$$\begin{aligned}\mathbb{P}(w_t < \tau \mid w_{t-1}, \varepsilon^m_{t-1}) &= \mathbb{P}(\alpha w_{t-1} + \eta_t < \tau \mid w_{t-1}, \varepsilon^m_{t-1}) \\ &= \mathbb{P}(\alpha w_{t-1} + \rho'_m \varepsilon^m_{t-1} < \tau \mid w_{t-1}, \varepsilon^m_{t-1}).\end{aligned}$$

When $\alpha > 0$, this probability reduces to the indicator function

$$\mathbb{P}(w_t < \tau \mid w_{t-1}, \varepsilon^m_{t-1}) = \mathbf{1}\Big\{\alpha w_{t-1} + \rho'_m \varepsilon^m_{t-1} < \tau\Big\}.$$

Case $s_{t-1} = 0$**.** Conditioning on $w_{t-1} < \tau$, we obtain

$$\begin{aligned}\omega_\rho(s_{t-1} = 0, \varepsilon^m_{t-1}) &= \mathbb{P}(\alpha w_{t-1} + \rho'_m \varepsilon^m_{t-1} < \tau \mid w_{t-1} < \tau, \varepsilon^m_{t-1}) \\ &= \begin{cases} 1, & \text{if } \frac{1}{\alpha}(\tau - \rho'_m \varepsilon^m_{t-1}) < \tau, \\ \dfrac{\Phi\Big((\tau - \rho'_m \varepsilon^m_{t-1}) \frac{\sqrt{1-\alpha^2}}{\alpha}\Big)}{\Phi(\tau\sqrt{1-\alpha^2})}, & \text{otherwise.} \end{cases}\end{aligned}$$

Case $s_{t-1} = 1$**.** Similarly, conditioning on $w_{t-1} \geq \tau$ yields

$$\omega_\rho(s_{t-1} = 1, \varepsilon^m_{t-1}) = \frac{\Phi\Big((\tau - \rho'_m \varepsilon^m_{t-1}) \frac{\sqrt{1-\alpha^2}}{\alpha}\Big) - \Phi(\tau\sqrt{1-\alpha^2})}{1 - \Phi(\tau\sqrt{1-\alpha^2})} \mathbf{1}\left\{\frac{\tau - \rho'_m \varepsilon^m_{t-1}}{\alpha} \geq \tau\right\}.$$

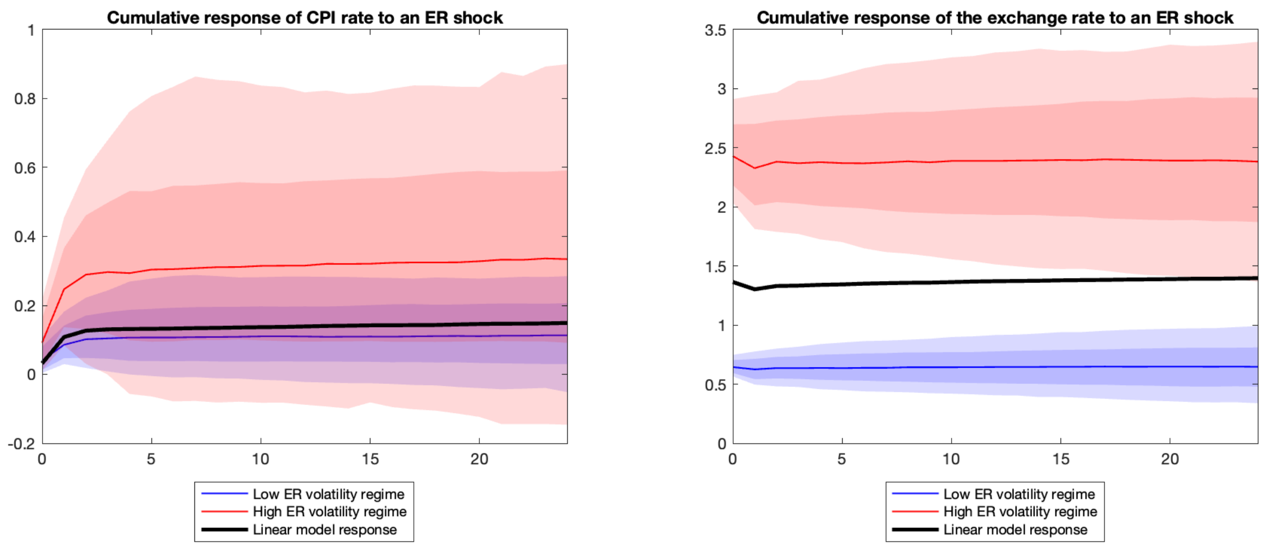
□

B Impulse Response Functions from Alternative Specifications

This appendix reports the impulse response functions (IRFs) corresponding to the alternative specifications considered in the robustness analysis. In all cases, we focus on the response to an identified exchange rate shock and report the implied responses of the exchange rate and CPI inflation. The identification strategy follows the recursive (Cholesky) ordering described in the main text, with the ordering adjusted as indicated for each alternative specification.

Unless otherwise noted, specifications use two lags in the VAR and the same horizon length as in the benchmark model. Differences across figures therefore reflect changes in the information set, lag length, data construction, or sample period rather than changes in the

Figure 12: Impulse responses from the expanded seven-variable specification (one lag). The system includes global oil prices, U.S. inflation, the shadow federal funds rate, domestic activity, the exchange rate, the domestic policy rate, and CPI inflation.



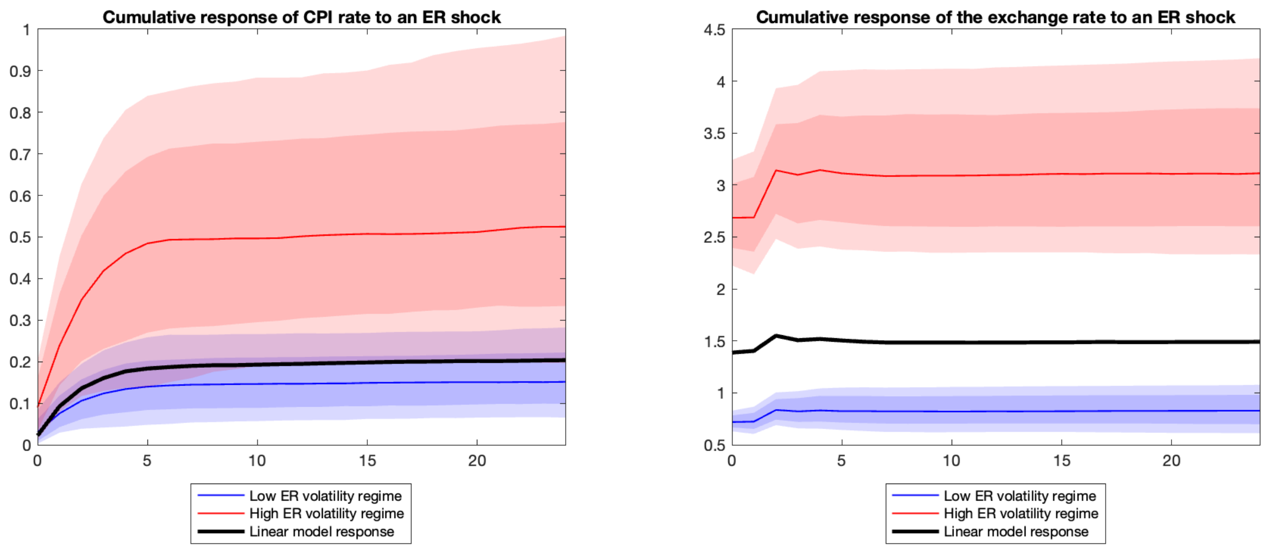
identification strategy.

B.1 Exchange Rate and Inflation IRFs

Expanded Information Set (7 Variables, 1 Lag). Figure 12 reports IRFs from the expanded system that augments the baseline four-variable specification with U.S. monetary policy (shadow federal funds rate), domestic economic activity, and the domestic policy rate. Real variables enter in log differences, while policy rates enter in levels. The VAR is estimated with one lag. The figure shows that incorporating external and domestic monetary conditions does not materially alter the qualitative pattern of exchange rate dynamics or the inflation response.

External Monetary Controls (Oil + U.S. Policy Rate). Figure 13 presents IRFs from a parsimonious four-variable system that includes global oil prices, the shadow federal funds rate, the exchange rate, and CPI inflation. This specification isolates external cost and monetary conditions while excluding domestic activity and policy variables. The exchange rate shock and the inflation response remain qualitatively similar to those in the benchmark specification.

Figure 13: Impulse responses from the four-variable specification including oil prices and the shadow federal funds rate.



Costa Rica Rate-Based Specification. Figure 14 reports IRFs from a specification that uses Costa Rica–constructed rate measures directly rather than log differences. The system includes the Central Bank oil price index rate, trading-partner inflation, the monthly percent change in the nominal exchange rate, and monthly CPI inflation. This alternative confirms that the results are not driven by the use of log transformations.

Restricted Sample. Figure 15 displays IRFs estimated over a shorter subsample. The information set and ordering are unchanged relative to the baseline; only the estimation window differs. The similarity of the responses indicates that the main findings are not driven by the inclusion of later observations.

Expanded System with Alternative Domestic Ordering (1 Lag). Figure 16 reports IRFs from the expanded seven-variable system under an alternative recursive ordering in which the exchange rate precedes domestic activity. This allows exchange rate innovations to affect activity contemporaneously within the month. The VAR is estimated with one lag. The qualitative behavior of the exchange rate and inflation responses remains consistent with the benchmark ordering.

Figure 14: Impulse responses from the Costa Rica rate-based specification using domestically constructed rate measures.

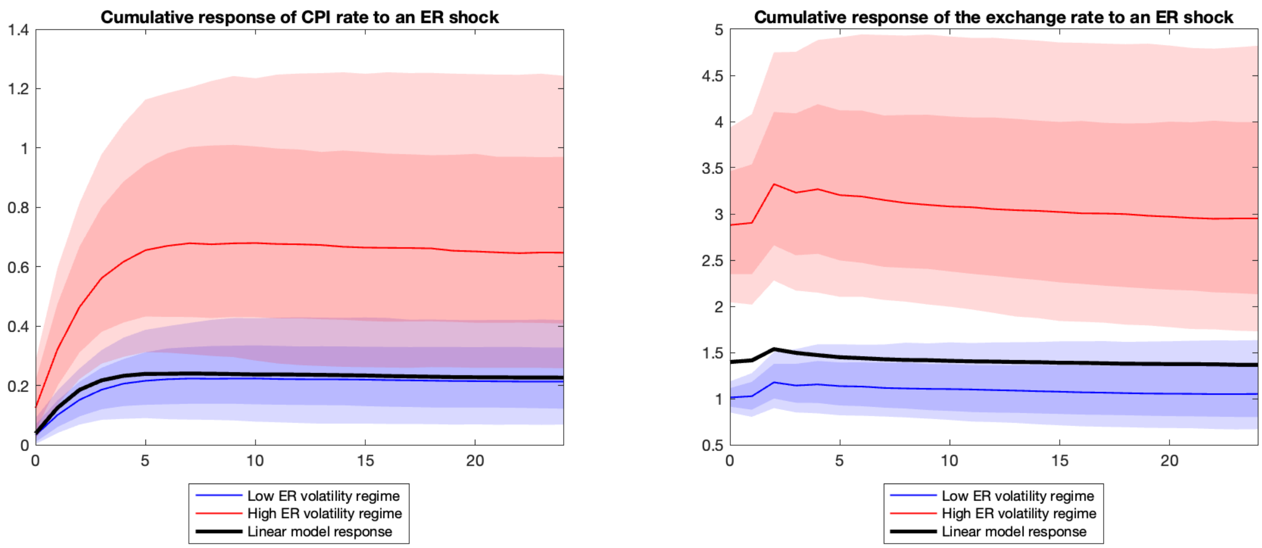


Figure 15: Impulse responses from the restricted-sample specification.

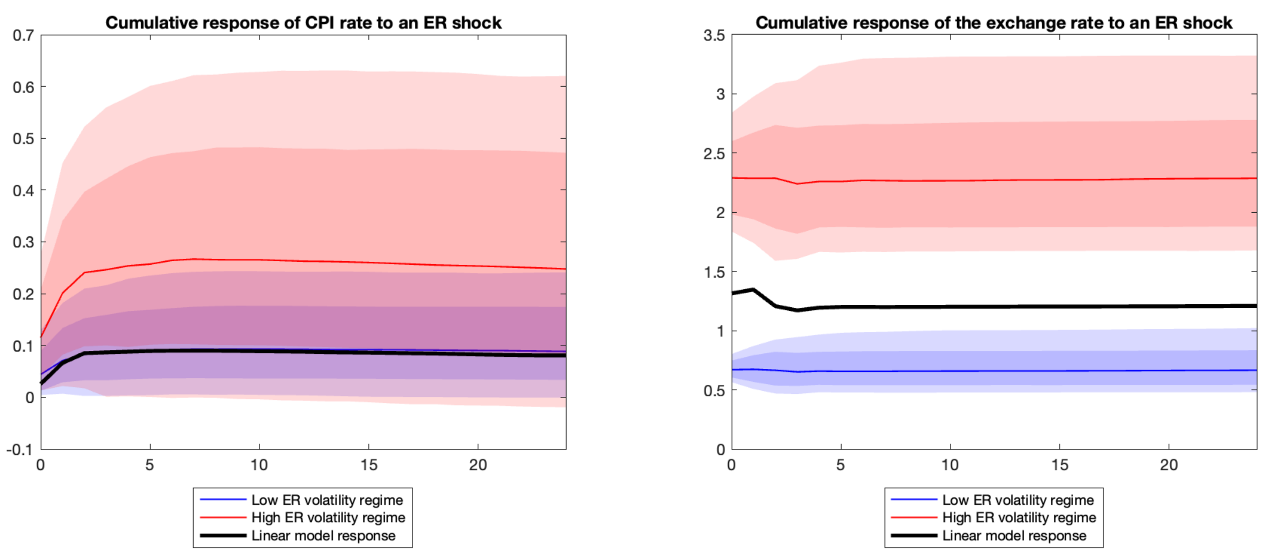


Figure 16: Impulse responses from the expanded seven-variable system under an alternative domestic ordering.